MATHEMATICAL MODELS IN OPTIMIZING AGRICULTURAL PRODUCTION

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RESEARCH ARTICLE

Abstract
This paper refers to the application of the simplex method in agriculture, specifically in optimizing agricultural resources. In the context of a farm, there are several resources (budget, water, land, time) available that need to be managed to maximize profit. The goal is to use the simplex method to allocate resources optimally to maximize profit. Mathematics provides essential tools for optimizing agricultural processes. The correct application of models can help farmers make informed decisions, saving resources and maximizing profits.

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INTRODUCTION

The importance of mathematics in agriculture in this paper is:

1.Use of models for production forecasting,

2.Cost-benefit analysis for crops,

3.Irrigation planning and resource management.

The case study is optimizing crop rotation. We analyze the problem: a farmer has 10 hectares of land on which he wants to grow wheat, corn and soybeans. The available resources are:

- Budget: 20000 RON,
- Water for irrigation: 50000 liters,
- Available working time: 1000 hours,

We have four types of resources that are used to grow two types of agricultural products: corn and wheat. Each crop requires a specific amount of resources (budget, water, land, and time), and the objective is to maximize the farmer's profit.

The problem statement is presented above and follows the Table 1 table containing the required resources and profit per hectare.

Table 1

Required resources and profit per hectare

Crop	Cost per hectare (RON)	Water required per hectare (liters)	Working time per hectare (hours)	Profit per hectare (RON)
Grow	2000	4000	120	3000
Corn	2500	5000	150	4000
Soybeans	1800	3000	100	2500

Solving the problem leads to the use of the simplex method or dedicated software (ex. Excel Solver, MATLAB, Python). The simplex method is a technique used in linear optimization to solve maximization or minimization problems, under linear constraints. In the case presented, we want to maximize profit. The optimal result indicates the distribution of the land for maximizing profit, respecting the constraints.

MATERIAL AND METHOD

The mathematical model we will be described as follows: we maximize the profit P under constraints:

$P = 3000\ X_1+4000\ X_2+2500\ X_3$

1. $2000\ X_1+2500\ X_2+1800\ X_3 \leq 20000$ (budget)

2. $4000 X_1+5000 X_2+3000 X_3 \le 50000$ (water)
3. $120 X_1+150 X_2+100 X_3 \le 1000$ (time of work)
4. $X_1+ X_2+ X_3 \le 10$ (available land)
5. $X_1, X_2, X_3 \ge 0$.

We will detail the steps of the Simplex method applied to the problem of crop rotation optimization in the previous example, we start from the formulated problem and illustrate how the method works.

In this linear optimization problem objective function is P which will be maximized, where X_1, X_2, X_3 represents the hectares cultivated with wheat, corn and soybeans.

In the simplex method, slack variables are introduced to transform the constraints into equality equations. In our case, the slack variables s_1, s_2, s_3, s_4 are added to represent the unused budget, water, labor time, and land.

Add slack variables (s_1, s_2, s_3, s_4) to turn constraints into equality:

$2000 X_1+2500 X_2+1800 X_3+ s_1 = 20000$
$4000X_1+5000X_2+3000X_3 + s_2 = 50000$
$120 X_1+150 X_2+100 X_3 + s_3= 1000$
$X_1+ X_2+ X_3 + s_4 = 10$

We organize the problem in the form of a simplex table 2.

Table 2

Assembling the Initial Simplex Table

Basic variables	x_1	x_2	x_3	s_1	s_2	s_3	s_4	Solution
s_1	2000	2500	1800	1	0	0	0	20000
s_2	4000	5000	3000	0	1	0	0	50000
s_3	120	150	100	0	0	1	0	1000
s_4	1	1	1	0	0	0	1	10
Z	-3000	-4000	-2500	0	0	0	0	0

The objective function will have the form $Z = 0-3000 X_1 -4000 X_2-2500 X_3$.

We identify the most negative value in the last line of Z basic variable, to select the pivot column. In this case, X_2 has the lowest coefficient, -4000.

The determination of the pivot row follows as follows. We calculate the ratio of the current solution to the positive coefficient in the pivot column. We select the row with the minimum ratio. We have ratios: 20000/2500=8; 50000/5000=10; 1000/150=6,67; 10/1=10.

We will transform the pivot element 150 in 1, thus by dividing row s_3 to 150, then adjusting the other rows so that all the other values in the pivot column X_2 become 0.

The pivot row is the third for s_3. We transform the table so that the pivot element becomes 1 and the rest of the elements in the pivot column becomes 0. We continue with this process until there are no more negative values in line Z.

The table is updated by transforming the pivot row like this: we divide the pivot row (s_3) the pivot (150):

new s_3 = 150/old s_3.

Updating the rest of the table we use the updated pivot row to make the rest of the elements in column X_2 equal to 0.

We apply the rule:

new line = old line − (pivot coefficient x pivot line).

To avoid errors and calculate the intermediate tables exactly, I will automate the calculation process and generate all the tables step by step. The calculations start now.

Updating the simplex table after pivoting on X_2, the simplex table 3 looks like this:

Table 3

Simplex Table Update

Basic var.	X_1	X_2	X_3	s_1	s_2	s_3	s_4	Solution
s_1	0	0	133,33	1	0	-16,67	0	3333,33
s_2	0	0	-333,33	0	1	-33,33	0	16666,67
x_2	0,8	1	0,67	0	0	0,01	0	6,67
s_4	0,2	0	0,33	0	0	-0,01	1	3,33
Z	200	0	166,67	0	0	26,67	0	26666,67

Interpretation after the first step: the Z-line no longer has negative coefficients, which indicates that we have found the optimal solution.

Final solution:

$X_1 = 0$
$X_2 = 6,67$
$X_3 = 0$

Maximum profit: P = 26666,67 RON.

In the simplex method, the variables s_1, s_2, s_3, s_4 are slack variables. They are added to the system of equations to convert the initial constraints (inequalities) into equations. The role of slack variables when we have form constraints:

$a_1 X_1+a_2 X_2+a_3 X_3 \le b$.

We add a slack variable (s) so that we transform inequality into an equation:

$a_1 X_1 + a_2 X_2 + a_3 X_3 + s = b$. The slack variable is the resource left unused after we allocated as much as we could to the decision variables (X_1, X_2, X_3).

In our context: s_1, s_2, s_3, s_4 variables mean s_1**:**

Budget left unspent:
- Initial constraint:
  $2000 X_1+2500 X_2+1800 X_3 \leq 20000$,
- After the allocation of crops, s_1 shows how much budget is left unused,

Water left unused:
- Initial constraint:
  $4000 X_1 + 5000 X_2 + 3000 X_3 \leq 50000$,
- s_2 represents the water that is not consumed by the chosen crops,

Unused working time:
- Initial constraint:
  $120 X_1 + 150 X_2 + 100 X_3 \leq 1000$,
- After the land is allocated, s_3 shows how much time is available,

The land left unused:
- Initial constraint:
  $X_1+ X_2+ X_3 \leq 10$,
- s_4 indicates how much land is not used for any crop.

RESULTS AND DISCUSSIONS

After applying the simplex method step by step, we get the optimal solution: land allocation: X_1 =a hectares grow, X_2 = b hectares corn, X_3 = c hectares soybeans.

The optimal solution is land allocation:
X_1 = 0 hectares grow, X_2 = 6,67 hectares corn,
X_3 = 0 hectares soybeans. Maximum profit: P = 26667,67 RON.

Interpretation: the farmer should allocate all available resources (budget, water, time) to grow corn on 6,67 hectares. The rest of the land (3,33 hectares) remains unused, as the cultivation of other crops would not maximize the profit within the imposed limits.

The slack variable is the resource left unused after we allocated as much as we could to the decision variables X_1, X_2, X_3. Thus, s_1 shows how much budget is left unused, s_2 represents the water that is not consumed by the chosen crops, s_3 shows how much time is available and s_4 indicates how much land is not used for any crop.

At the end of the calculation, if a slack variable (s_1, s_2, s_3, s_4) has the value 0, means that the resource has been fully used. If the slack variable has a positive value, that resource hasn't been fully utilized. In our case, the optimal solution shows that: part of the land (s_4 = 3,33) remains unused. The rest of the slack variables will show how the other resources were consumed. Here's how to interpret the values of slack variables (s_1, s_2, s_3, s_4) in the final table 4:

Table 4

Interpret the values of slack variables

Variables	Meaning	Final value	Interpretation
s_1	Unspent budget	3333,33	Of the total budget of 20000 RON, 3333,33 RON remains unused.
S_2	unused water	16666,67	Of the total 50000 liters of water available, 16666,67 liters are not consumed.
S_3	unused working time	0	All available time (1000 hours) is fully utilized.
S_4	Unused land	3,33	Of the 10 hectares available, 3,33 hectares remain unused.

CONCLUSIONS

Detailed interpretation: budget (s_1) the farmer spends 16666,67 RON on crops and is left with a surplus of 3333,33 RON, water (s_2) resources are used in a proportion of 3333,33 liters for the optimal crop (corn on 6,67 hectares), and the remaining 16666,67 liters remain available, time (s_3) all the 1000 hours available are consumed for agricultural work, without any surplus, land (s_4) after allocating 6,67 hectares for corn crops, the farmers still have 3,33 hectares of unused land.

The optimal solution maximizes the profit to 26666,67 RON, but leaves some resources (budget, water, land) unused. This suggests that other resources (such as working hours) are more restrictive constraints for this optimization.

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