ANALYTICAL STUDY OF DISPLACEMENTS AND DEFORMATIONS OF WOODEN BEAMS USING THE METHOD OF INITIAL PARAMETERS

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Abstract

The work presented and conceived by the author is intended to be a starting point in all calculations of strength and rigidity. It can be considered as a novelty in the case of these types of problems through the calculation program designed by the author. The paper briefly presents the analytical study of the problem, focusing on numerical analysis. The constructive element considered is made of wood, but can be adapted for any other type of material.

Key words: analitycal, numerical, bar, kinetics, parameters.

INTRODUCTION

Practical engineering experience has often confirmed the importance of the behavior and safe operation of the components of mechanical or civil structures. As it is known, the safety of operation is largely related to the calculations of strength and rigidity in execution projects.

The mechanical analysis of the different elements within the different constructive or mechanical structures also implies an approach to the problems related to their rigidity calculation. The present paper wishes to present in detail the practical analytical and numerical application of the method of initial parameters applied to an element of wood material component of a structure. The author considered in the paper an element made of beam-type wood, with a circular section of constant diameter and its length. The initial data of the presented paper are the following:

- l, beam length;
- E, Young's module;
- I, the moment of axial inertia;
- σ_{adm}, the allowable tension of the material;
- v_{adm}, maximum allowed arrow;
- φ_{adm}, the permissible rotation of the beam sections.

MATERIAL AND METHOD

Regarding the admissible arrow, it is chosen depending on the use and the place where the constructive element is located.

In the case of the initial parameter's method, it is sought to determine the general expressions of the kinematic parameters represented by displacements and rotations, as well as of the static parameters represented by the bending moments and the shear forces.

Unlike the direct integration method, the method presented and applied in the paper presents a much simpler calculation algorithm compared to the direct integration method even for a small number of intervals of variation of the bending moment, the need to determine the integration constants by writing is eliminated. boundary and boundary conditions between intervals (Catarig, Kopenetz, 2001, Ciofoaia, Curtu, 1986)

The case study was performed on the bar element loaded with a system of equilibrium forces composed of a uniformly distributed force, a concentrated force and a bending moment.

The following notations have been adopted

q, intensity of the force evenly distributed;

F, concentrated external force;

The constructive element is considered in the paper as simply supported, it being part of the statically determined systems. The study starts from the mechanical analysis of the original parameters with the prior determination of the connection forces from the supports of the considered constructive element. The kinetics allowable values chosen by the author are:

- $v_{adm} = \frac{l}{600}$ [mm]
- $\varphi_{adm} = 1°$
- $\sigma_{adm} = 120 \left[\frac{N}{mm^2}\right]$
- the moment of axial inertia of the section, I = (π · D^4)/64 [m^4];
- the allowable arrow was considered to be, L/1000 = 2 [mm]
- allowable rotation, φadm = 1°
- uniformly distributed load having intensity q = ρ · g≅50 [kN / m].

The study tries to present besides the application of the method of the initial parameters and a way of its numerical application for the case of the considered constructive elements.

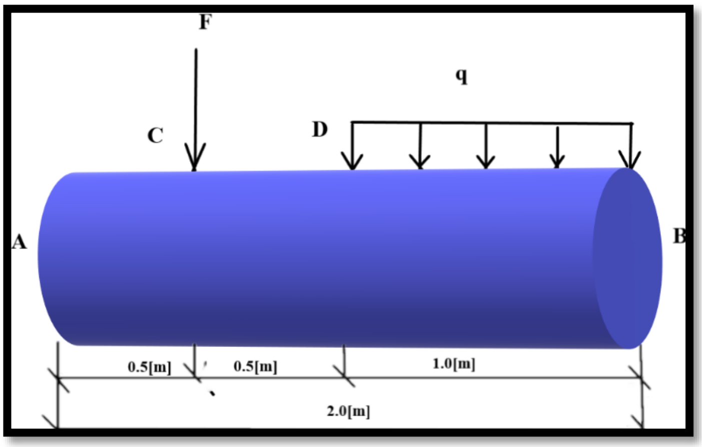

Fig. 1 Geometric element loaded with external forces

The practical analytical and numerical study was performed in the XOY plane, considering the constructive element represented only by the bar axis.

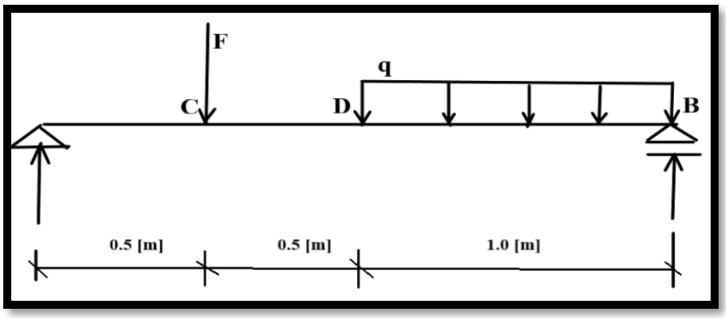

Fig. 2 The constructive element represented in the plane by its own longitudinal axis

RESULTS AND DISCUSSION

The application of the calculation algorithm of the initial parameters method led to the following results (Fetea, 2010), (Goia, 2000), (Ille., 1981)

$$\sum M_A = 0;$$

$$\sum M_B = 0;$$

$$-2V_B + 2.5q + 0.5F = 0$$

$$2V_A - 0.5q - 1.5F = 0$$

$$\mathrm{V_B} = \frac{(2.5\mathrm{q} + 0.5\mathrm{F})}{2}$$

$$V_A = \frac{(0.5q + 1.5F)}{2}$$

Taking into account the mode of support and loading of the element, the initial parameters identified are the following (Gheorghiu, Hadar, 1998):

$$v_0 = 0;$$

$$\varphi_0 \neq 0;$$

$$M_0 = 0;$$

$$T_0 = V_A = \frac{(0.5q+1.5F)}{2}.$$

Having the initial parameters partially determined the general equation of the arrow becomes (Ivan M. Statica, 1997), (Marțian, 1999):

$$v(x) = \varphi_0 x - \frac{T_0 x^3}{6EI} - \frac{F(x - 0.5)^3}{6EI} + \frac{q(x - 1)^4}{24EI}$$

The rotation of the body axis is non-zero and unknown. To determine it, the author imposed the limit condition (Missir-Vlad, 2002):

$$v(2) = 0 \rightarrow \varphi_0$$

$$v(2) = 0$$

$$\varphi_0 = \frac{\frac{T_0 x^3}{6EI} + \frac{F(x - 0.5)^3}{6EI} - \frac{q(x - 1)^4}{24EI}}{2000}$$

The determined value of the rotation φ_0 is introduced in the general equation of the displacements, obtaining the final expression of their determination. By convenient derivations were obtained the expressions of rotation, respectively of the bending moment and of the shape shear force taking into account the differential equation of the deformed axis Missir-(Vlad, Ioana, 2002):

$$\frac{d^2v}{dx^2} = -\frac{M_X}{EI}$$

$$\varphi(x) = \varphi_0 - \frac{T_0 x^2}{2EI} - \frac{F(x - 0.5)^2}{2EI} + \frac{q(x - 1)^3}{6EI}$$

Having the general expressions of the kinematic and static parameters of the cross sections of the considered constructive element, their values can be determined in any element section. For the case studied. a numerical calculation program was designed in Matlab, aiming to determine the same parameters as in the case of the analytical study of the beam (Muntenu Gh.,1998). The following calculation program designed by the author and named – **The calculation program of the kinematic parameters.**

```
% TITLE - The calculation program of the kinematic parameters
% D, diameter
% E, Young's modulus
% I, axial inertia moment
% F, External force
% L, lenght bar
% q, load intensity
% VA, force reaction in A section
% v, general expression of displacements
% ROT0, initial spin
% ROT, general expression of spin
% v1,v2,v3, displacements by intervals
% rot1,rot2,rot3, spin by intervals
D=400
E=2.1*10^5
I=(pi*D^4)/64
q=50
F=100
L=2000
VA=87.5
ROT0=((87.5*10^3*2^3)*(10^9)/(6*2.1*10^5*1.256*10^9)+(100*(1.5)^3)
*(10^12)/(6*2.1*10^5*1.256*10^9)-
(50*(1)^4)*(10^12)/(24*2.1*10^5*1.256*10^9))/2000
syms x E I
v=((ROT0*x)*10^3)-(((VA*x^3)*10^12)/(6*E*I))-(((F*(x-
0.5)^3)*10^12)/(6*E*I))+(((q*(x-1)^4)*10^12)/(24*E*I))
ROT=diff(v)
M=-(diff(ROT))*(E*I);
T=diff(M);
for x=[0 0.1 0.2 0.3 0.4 0.5 ];
   v1=((ROT0*x).*10^3)-(((87.5*x.^3)*10^12)./(6*2.1*10^5*I))
end
for x=[0.6 .7 .8 .9 1];
  v2=((ROT0*x).*10^3)-(((87.5*x.^3).*10^12)./(6*2.1*10^5*I))-(((100*(x-
0.5).^3).*10^12)./(6*2.1*10^5*I));
end;
for x=[1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2];
```

```
  v3=((ROT0*x).*10^3)-(((87.5*x.^3).*10^12)./(6*2.1*10^5*I))-(((100*(x-
0.5).^3).*10^12)./(6*2.1*10^5*I))+(((50*(x-
1).^4).*10^12)./(24*2.1*10^5*I));
end;

syms x VA E I
v1=((ROT0*x)*10^3)-(((VA*x^3)*10^12)/(6*E*I))
rot1=diff(v1)
for x=[0 0.1 0.2 0.3 0.4 0.5 ]
   for VA=87.5
      for E=2.1*10^5
         for I=1.256*10^9
   rot1=746729692537419125/2305843009213693952                              -
(500000000000*VA*x.^2)./(E*I)
         end
      end
   end
end

syms x VA E I
v2=((ROT0*x).*10^3)-(((87.5*x.^3).*10^12)./(6*2.1*10^5*I))-(((100*(x-
0.5).^3).*10^12)./(6*2.1*10^5*I))
rot2=diff(v2)
for x=[0.6 .7 .8 .9 1]
   for VA=87.5
      for E=2.1*10^5
         for I=1.256*10^9
   rot2=746729692537419125/2305843009213693952                              -
(1127428915200000000000000*x.^2)./(5411658792960001*I)                      -
(1288490188800000000000000*(x - 1/2).^2)./(5411658792960001*I)
         end
      end
   end
end

syms x VA E I
v3=((ROT0*x).*10^3)-(((87.5*x.^3).*10^12)./(6*2.1*10^5*I))-(((100*(x-
0.5).^3).*10^12)./(6*2.1*10^5*I))+(((50*(x-
1).^4).*10^12)./(24*2.1*10^5*I))
rot3=diff(v3)
for x=[1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2]
```

```
   for VA=87.5
      for E=2.1*10^5
         for I=1.256*10^9
   rot3=(2147483648000000000000000*(x - 1).^3)./(5411658792960001*I) - (12884901888000000000000000*(x - 1/2).^2)./(5411658792960001*I) - (11274289152000000000000000*x.^2)./(5411658792960001*I) + 746729692537419125/2305843009213693952
         end
      end
   end
end
```

The determined values of the arrows and of the rotations in the body sections are:

Displacements	Rotations	Sections coordinates
v1 = 0	rot1 = 0.3238	for x=0
v1 =0.0323	rot1 = 0.3222	for x=.1
v1 =0.0643	rot1 = 0.3172	for x=.2
v1 =0.0957	rot1 = 0.3089	for x=.3
v1 =0.1260	rot1 = 0.2973	for x=.4
v1 =0.1550	rot1 = 0.2824	for x=.5
v2 =0.1823	rot2 =0.2622	for x=.6
v2 =0.2072	rot2 =0.2350	for x=.7
v2 =0.2291	rot2 =0.2006	for x=.8
v2 =0.2471	rot2 =0.1592	for x=.9
v2 =0.2607	rot2 =0.1106	for x=1
v3 =0.2690	rot3 =0.0549	for x=1.1
v3 =0.2714	rot3 =-0.0076	for x=1.2
v3 =0.2672	rot3 =-0.0769	for x=1.3
v3 = 0.2558	rot3 =-0.1528	for x=1.4
v3 =0.2365	rot3 =-0.2350	for x=1.5
v3 =0.2086	rot3 =-0.3233	for x=1.6
v3 =0.1716	rot3 =-0.4177	for x=1.7
v3 =0.1249	rot3 =-0.5178	for x=1.8
v3 =0.0679	rot3 =-0.6235	for x=1.9
v3 =3.8164e-17	rot3 =-0.7346	for x=2

CONCLUSIONS

The conclusions that can be drawn from this study are the following:

1. The elaboration of this program by the author represents an element of novelty, which allows solving any problem of static calculation

of wooden beams, regardless of the external forces acting, the type of wood material, its mechanical-physical characteristics. The problem is easy to solve just by changing the values in the program. This feature will make the work of any engineer easier, being necessary to apply only the calculation program used.

2. The program made by the author allows the exact determination of the kinematic parameters in any of the sections of the constructive element.
3. The program allows the determination of displacements and rotations for any constructive element of bar type, regardless of the type of material, the external forces and the considered supports.
4. The numerical analysis through the conceived program, leads to exact and correct results regarding the displacements and rotations according to the supports of the constructive element.
5. The practical implementation of numerical calculation methods is certainly in the future the only viable methods in quickly and accurately solving computational problems in engineering.

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