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STUDY REGARDING THE IMPLEMENTATION OF THE MATRIX MATERIAL CALCULATION PROGRAMS IN DETERMINING THE DEFORMATIONS OF THE BENDED BEAMS

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Abstract

The analysis of the mechanical behavior of different wooden structures in order to determine the safety limits in their operation of wood was, is and will remain one of the reference concerns of engineers in their various fields of applicability. Regardless of the type of structure studied (beam, plate, or lattice beam), the problems are related to ensuring safety rules depending on the loads imposed on them. It was found that the application of analytical calculation methods requires increased attention and a rather laborious mathematical apparatus and also does not ensure sufficiently rigorous results. Thus, the paper tries to present the elaboration of a numerical calculation program of the displacements by the method of direct integration using the Matlab program.

Key words: analitycal, numerical integration, beam, deflections.

INTRODUCTION

This paper presents a comparative analysis on determining the displacements of the main sections of a bar with a circular section (arrows and rotations) using the method of direct integration of the differential equation of equilibrium of the deformed axis applied analytically and numerically. For the analytical and numerical analysis of the displacements from the main sections of the bar, the same bar was considered having the same supports, external loads, the same cross section as shape and size. For both cases the stiffness of the bending bar is considered to be the constant $E \cdot I = const.$

Analytically, we will start from the calculation of the binding forces, of the efforts, the dimensioning and the verification of the bar, after which the displacements and rotations of the main sections will be determined using the direct integration method (Marţian, 1999), (Fetea, 2010). The comparison was made with the same method but its application was done numerically using matrix calculus (Muntenu, 1998). Thus each static or kinematic parameter, respectively dimensional element will be written in the form of a vector or matrices with $n \times m$ rows and columns.

The studied wooden bar has the following characteristics:

- the length of the bar is 3.0 [m];
- the beam section is circular with initially unknown diameter "D";

- the beam is considered to be embedded in the structure of a roof of a family home;
- the permissible calculation stress for oak parallel to the fibers at a humidity of 12% is, σadm = 15 [N /mm ^ 2];
- the longitudinal modulus of elasticity for humidity of 12% is E = 0.1 * 10 ^ 5 [N / mm ^ 2];
- density of wood material (oak), ρ = 700 [Kg / m ^ 3];
- the moment of axial inertia of the section, I = (π · D^4)/64 [m^4];
- the allowable arrow was considered to be, L/1000 = 2 [mm]
- allowable rotation, φ_adm = 1°
- uniformly distributed load having intensity q = ρ · g≅20 [kN / m].

MATERIAL AND METHOD

The analytical study of the considered bar presents the following calculation algorithm:

- the connecting forces in the supports were determined;

- the sectional efforts (bending moments and shear forces) from the main sections of the bar were determined;

- the most requested cross section was dimensioned starting from the resistance condition to the written bending stress in normal stresses;

- the most requested section was checked;

- using the direct integration method, the expressions of displacement and rotation were determined on the variation intervals of the bending moment (Ciofoaia E., 2001), (Ivan M.,1997);

- the maximum arrow and the maximum rotation are checked depending on the admissible arrow and the admissible rotation for the studied case;

$$\sum M_A = 0$$

$$-V_B L + q\frac{L^2}{2} = 0$$

$$V_B = q\frac{L}{2}$$

$$V_B = V_A = 20[KN]$$

The analysis follows the classic stages of mechanical calculation of the bars, determining according to the calculation algorithm the following parameters:

The efforts in the main sections beam (Catarig., 2001), (Missir V., 2002):

$$M_B = M_A = 0[KNm]$$
$$M_C = 10[KNm]$$

The shear forces in A and B sections

$$T_A = V_A = 20[KN]$$
$$T_B = V_B = -20[KN]$$

The dimensioning of the dangerous section will be done using the resistance condition in normal mechanical stresses (Catarig., 2001), (Ille V., 1981)

$$\sigma_{max} = \frac{M_{max}}{W_{nec}} \leq \sigma_{adm}$$
$$\downarrow$$
$$D_{nec} = 189.7[mm]$$
$$D_{ef} = D_{nec} + 5 = 194.7[mm]$$

Checking the dangerous section of the beam in normal mechanical stresses leads to the result (Missir V., 2002):

$$\sigma_{max} = \frac{M_{max}}{W_{ef}} = 13.87[\frac{N}{mm^2}] \leq \sigma_{adm}$$

To determine the expressions of rotation and displacement along the axis of the bar, the method of direct integration of the differential equation of the deformed axis of the bar was used (Hadar A., 1998). Starting from the expression of the bending moment along the beam axis, written according to the independent parameter "x", by two successive integrations the expressions of rotation and displacements were determined. The expressions do not allow their direct determination being a function of two integration constants (Catarig., 2001) , (Goia I., 2000). By imposing the conditions at the limits, the two integration constants C2 = 0 and C1 = 6.66 were determined

$$\varphi = \frac{dv}{dx} = -\frac{1}{EI}\int\left(V_A x - q\frac{x^2}{2}\right)dx + C_1$$
$$v = \frac{d\varphi}{dx} = -\frac{1}{EI}\int(V_A\frac{x^2}{2} - q\frac{x^3}{6}dx + C_1)dx + C_2$$

Imposing for x=0 [m] and x=2 [m] result:

$$C_1 = 6.66$$

$$C_2 = 0$$

RESULTS AND DISCUSSION

For the case studied. a numerical calculation program was designed in Matlab, aiming to determine the same parameters as in the case of the analytical study of the beam (Muntenu Gh.,1998). The following calculation program designed by the author and named - deflectionbeamssolve -MSF was made.

```
% Name program – “Deflectionbeamssolve – MSF”
% Effort study

% L – beam length [m]

% q - uniformly distributed load intensity [KN/m^2]
% VA, VB - forces
% R, mechanical resulting force of uniformly distributed force

% Vadm, allowable displacement[mm]

Sigmaadm=15

L =0:.2:2
Vadm=(L(1,11)*10^3)./1000
q=20
% Determination of sectional efforts

% Shear forces - TA, TB

R=20*L(1,11)
VA=R./2
VB=VA
TA=VA
TB=-VB
 syms T(x)
T(x) = VA-q*x
sol = vpasolve(T)
for x=L(1,6)
   T(x)
end
 TC=T(x)
T=[TA TC TB]
 % Bending moments
% MA, MB, MC –bending moments
MA=0
```

```
MB=0
syms M(x)
M(x)=VA*x-(q*x^2)/2
solMx = vpasolve(M)

for x=L(1,6)
   M(x)
end
MC=M(x)
 % Analysis of the mechanical strength condition

Sigmaadm=15
 syms x
 g(x)=x-((32*MC*10^6)/(pi*Sigmaadm))^(1/3)
 solgx=vpasolve(g)
Dnec=solgx
Def=Dnec+5
M=[MA MC MB]
% Mechanical determination of the diameter of the dangerous section.

Sigmaadm=15

% " Sigmaadm" Admissible mechanical stress in [N/mm^2]
% Def - the effective diameter of the dangerous section

% Verification of dangerous section

% W - the effective mechanical resistance module of dangerous section
Wef = (pi*Def^3)/32
Sigmamax = (MC*(10^6))/Wef
% Check the mechanical stress allowed by15 [N/mm^2]

% Beam stiffness calculation

% E – Young's modulus [daN/cm^2]
% I - moment of axial inertia [mm^4]

E=0.1*10^5
I=(pi*Def^4)/64
syms rot(x)
ode=diff(rot,x) == -(VA*x-(q*x^2)/2)
rotSol(x) = dsolve(ode)
x=0:.2:2
rot=6.66 + (10*x.^2.*(x - 3))./3
```

```
plot(x,rot)
 % Mechanical check of rotation in the middle of the beam opening= 0
for x=1
   rotSol(x)
end
C1=6.66
 % Determining the equation of displacement by integrating the first
derivative of the rotation, respectively the second derivative of the bending
moment.
syms v(x)

ode=diff(v,x) == C1 + (10*x^2*(x - 3))/3
vSol(x) = dsolve(ode)
 x=0:.2:2
v=((5*x.^4)/6 - (10*x.^3)/3 + x.*C1 + C2)
plot(x,v)

 for x=2
    for C1=6.66
       vSol(x)
    end
 end
C2=0
 % Determining the displacement equation by integrating the second order
differential equation of the deformed axis and imposing boundary
conditions on the ends of the bar for x = 0 and x = 2
syms v(x)
Dv = diff(v);
ode = diff(v,x,2) == -(VA*x-(q*x^2)/2)
cond1 = v(0) == 0;
cond2 = v(2) == 0;
conds = [cond1 cond2];
vSol(x) = dsolve(ode,conds)
vSol = simplify(vSol)
for C1=6.66
   for C2=0
      vSol(x)
   end
end
for x=1000
   (vSol(x)/(E*I))
end
```

```
% Mechanical determination of maximum rotation
rotation= C1 + (10*x^2*(x - 3))/3
for x=2000
    rotation/(E*I)
end
f1=rotation/(E*I)
% maximum rotation in degrees

maximumrotation=f1*360/(2*pi)

% Check the maximum rotation being 1 degree!
```

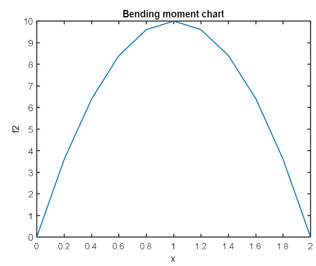

Fig.1.Bending moment chart

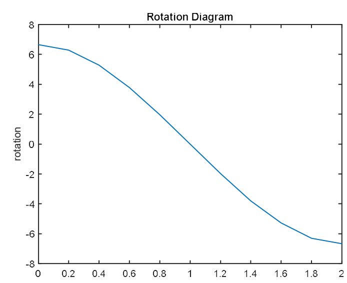

Fig. 2. Rotation diagram along the x-axis

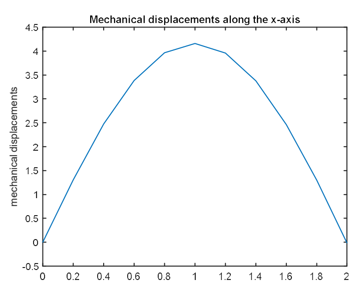

Fig. 3. Mechanical displacements along the x-axis

CONCLUSIONS

The conclusions that can be drawn from this comparative study are the following:

1. The elaboration of this program by the author represents an element of novelty, which allows solving any problem of static calculation of wooden beams, regardless of the external forces acting, the type of wood material, its mechanical-physical characteristics. The problem is easy to solve just by changing the values in the program. This feature will make the work of any engineer easier, being necessary to apply only the calculation program used.
2. As a conlusion, the „smart program" use to solve the deflections beams problem can be use in the light wood construction fields because will reduce significantly the time necessary to determine the correct values for the dimensioning of the section, its verification, the determination of the maximum displacements and rotations or in any section of the bar.
3. The practical implementation of numerical calculation methods is certainly in the future the only viable methods in quickly and accurately solving computational problems in engineering.

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