

## DYNAMIC ANALYSIS OF VIBRATIONS MODES FOR RECTANGULAR WOOD PLATES

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### Abstract

*This paper presents the modal analysis calculation of total and modal dynamical responses expressed in displacements and bending moments for the rectangular plate loaded with a uniformly distributed forces over the entire surface. Using the multiplier dynamic function is simplifies the dynamic methodology calculation of plates. The method considered the shapes functions of plates vibrations as the product of beams shapes functions having the same limits conditions. The results obtained in this paper reveal the most dangerous sections of the structure relative to the x and y axes.*

**Key words:** modal, wood, plate, vibration, shape, function.

### INTRODUCTION

The flat plate considered for the study is reported by a Cartesian coordinate system with the origin in the upper plate left corner. The plate has dimensions  $a$  along the x axis and  $b$  along the y axis. The aspect ratio considered for the studied plate is

$$\alpha = \frac{a}{b} = 1,5$$

In applying the modal analysis method were considered only symmetric modes of plate vibration plate  $i, j$ . For the clamped plate, the symmetrical shapes appear for the following normal modes of vibrations (1,1), (1,3), (3,1), (3,3). When performing modal analysis was taken into account the shapes functions expressions functions of the beams (Birsan, 1979), (Fetea, 2010), (Warburton, 1954), (Zienkiewicz, 1971) having the same limits conditions. Modal analysis was performed for clamped rectangular plate with a harmonic loading having pulsation

$$\Omega = 10 \frac{rad}{s}$$

The force is considered normal on the plate surface. The harmonic force has the expression (Birsan, 1979), (Birsan, 1971).

$$p(x, y, t) = p(x, y) \cdot f(t) \quad (1)$$

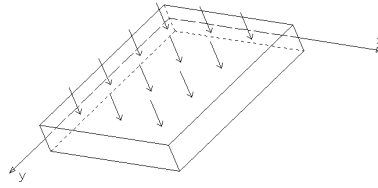


Fig.1 Plate load by uniform forces

Because of symmetry of plate was examined only defined domain by  $a = 3, b = 2$  (Figure 2).

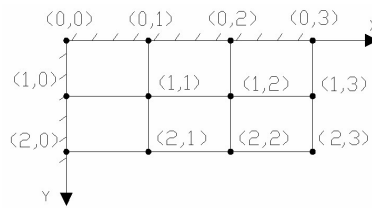


Fig.2 Nodes plate analysis

For this plate the bending moments on the edges along the  $x$  and  $y$  axes will have zero values. Applying the Galerkin-Vlasov method were determined the parameter pulsations for the plate ratio mentioned above.

#### MATERIAL AND METHOD

The own pulsations parameter values are calculated (table 1). The results obtained in (Birsan, 1979), (Fetea, 2010), (Lungu, 1990), (Soare, 1999).

Table 1

Pulsations parameters.

Modes	(1,1)	(1,3)	(3,1)	(3,3)
$\lambda_{ij}$	60,8	275,2	135,6	301,6
$\Omega_{ij}$	7,79	16,59	11,64	17,36

Starting from the static moments expressions in relation to  $x, y$  axes, we express displacement as given by the product of the beams shapes functions (Timoshenko, 1962), (Warburton, 1976) resulting the bending moments expressions:

$$M_{ij}^x(x, y) = -D \cdot \left( \frac{\partial^2 w_{ij}(x, y)}{\partial^2 x} - \nu \cdot \frac{\partial^2 w_{ij}(x, y)}{\partial y^2} \right); \quad M_{ij}^y(x, y) = -D \cdot \left( \frac{\partial^2 w}{\partial^2 y} - \nu \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

$$M_{ij}^x(x, y) = -D \cdot \left( \frac{\partial^2 [G_i(x) \cdot G_j(y)]}{\partial^2 x} - \nu \cdot \frac{\partial^2 [G_i(x) \cdot G_j(y)]}{\partial y^2} \right)$$

$$M_{ij}^y(x, y) = -D \cdot \left( \frac{\partial^2 [G_i(x) \cdot G_j(y)]}{\partial^2 y} - \nu \cdot \frac{\partial^2 [G_i(x) \cdot G_j(y)]}{\partial x^2} \right)$$

Developing the expressions by explaining the shapes functions of the beams (Birsan, 1979),(Fetea, 2010), is resulting the relations for static moments. They have the expression derivatives of order 2 in relation to the axes and the shapes functions of the bars are given by the expressions (Bors, 2005),(Fetea, 2010)

$$M_{ij}^y = -D \left\{ \left( \frac{\beta_j}{b} \right)^2 \left[ \left( \cosh \beta_j \cdot \frac{y}{a} + \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} + \sin \beta_j \frac{y}{b} \right) \right] \right\} \quad (2)$$

$$G_i^x(x) = \left( \frac{\beta_i}{a} \right)^2 \left[ \left( \cosh \beta_i \cdot \frac{x}{a} + \cos \beta_i \frac{x}{a} \right) - k_i \left( \sinh \beta_i \frac{x}{a} + \sin \beta_i \frac{x}{a} \right) \right] \quad (3)$$

$$G_j^y(y) = \left( \frac{\beta_j}{b} \right)^2 \left[ \left( \cosh \beta_j \cdot \frac{y}{b} + \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} + \sin \beta_j \frac{y}{b} \right) \right] \quad (4)$$

$$M_{ij}^x(x, y) = -D \cdot \left\{ \left( \frac{\beta_i}{a} \right)^2 \left[ \left( \cosh \beta_i \cdot \frac{x}{a} + \cos \beta_i \frac{x}{a} \right) - k_i \left( \sinh \beta_i \frac{x}{a} + \sin \beta_i \frac{x}{a} \right) \right] \cdot \left[ \left( \cosh \beta_j \cdot \frac{y}{b} - \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b} \right) \right] - v \cdot \left( \frac{\beta_j}{b} \right)^2 \left[ \left( \cosh \beta_j \cdot \frac{y}{b} + \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} + \sin \beta_j \frac{y}{b} \right) \right] \cdot \left[ \left( \cosh \beta_i \cdot \frac{x}{a} - \cos \beta_i \frac{x}{a} \right) - k_i \left( \sinh \beta_i \frac{x}{a} - \sin \beta_i \frac{x}{a} \right) \right] \right\} \quad (5)$$

$$M_{ij}^y(x, y) = \left( \frac{\beta_j}{b} \right)^2 \left[ \left( \cosh \beta_j \cdot \frac{y}{b} + \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} + \sin \beta_j \frac{y}{b} \right) \right] \cdot \left[ \left( \cosh \beta_i \cdot \frac{x}{a} - \cos \beta_i \frac{x}{a} \right) - k_i \left( \sinh \beta_i \frac{x}{a} - \sin \beta_i \frac{x}{a} \right) \right] - v \cdot \left( \frac{\beta_i}{a} \right)^2 \left[ \left( \cosh \beta_i \cdot \frac{x}{a} + \cos \beta_i \frac{x}{a} \right) - k_i \left( \sinh \beta_i \frac{x}{a} + \sin \beta_i \frac{x}{a} \right) \right] \cdot \left[ \left( \cosh \beta_j \cdot \frac{y}{b} - \cos \beta_j \frac{y}{b} \right) - k_j \left( \sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b} \right) \right] \quad (6)$$

Dynamic modal responses in bending moments are [1], [3]:

$$M_{ij}^x(x, y, t) = M_{ij}^x(x, y, t) \cdot \Psi_{ij} \quad M_{ij}^y(x, y, t) = M_{ij}^y(x, y, t) \cdot \Psi_{ij}$$

$$M_{11}^x(x, y) = -D \cdot \left\{ \left( \frac{\beta_1}{a} \right)^2 \left[ \left( \cosh \beta_1 \cdot \frac{x}{a} + \cos \beta_1 \frac{x}{a} \right) - k_1 \left( \sinh \beta_1 \frac{x}{a} + \sin \beta_1 \frac{x}{a} \right) \right] \cdot \left[ \left( \cosh \beta_1 \cdot \frac{y}{b} - \cos \beta_1 \frac{y}{b} \right) - k_1 \left( \sinh \beta_1 \frac{y}{b} - \sin \beta_1 \frac{y}{b} \right) \right] - v \cdot \left( \frac{\beta_1}{b} \right)^2 \left[ \left( \cosh \beta_1 \cdot \frac{y}{b} + \cos \beta_1 \frac{y}{b} \right) - k_1 \left( \sinh \beta_1 \frac{y}{b} + \sin \beta_1 \frac{y}{b} \right) \right] \cdot \left[ \left( \cosh \beta_1 \cdot \frac{x}{a} - \cos \beta_1 \frac{x}{a} \right) - k_1 \left( \sinh \beta_1 \frac{x}{a} - \sin \beta_1 \frac{x}{a} \right) \right] \right\} \cdot \Psi_{11} \quad (7)$$

$$M_{13}^x(x, y) = -D \cdot \left\{ \left( \frac{\beta_1}{a} \right)^2 \left[ \left( \cosh \beta_1 \cdot \frac{x}{a} + \cos \beta_1 \frac{x}{a} \right) - k_1 \left( \sinh \beta_1 \frac{x}{a} + \sin \beta_1 \frac{x}{a} \right) \right] \right\} \cdot \Psi_{13}$$



$$\begin{aligned}
& -\nu \cdot \left(\frac{\beta_3}{a}\right)^2 \cdot \left[ \left( \cosh \beta_3 \cdot \frac{x}{a} + \cos \beta_3 \frac{x}{a} \right) - k_3 \left( \sinh \beta_3 \frac{x}{a} + \sin \beta_3 \frac{x}{a} \right) \right] \cdot \\
& \left[ \left( \cosh \beta_1 \cdot \frac{y}{b} - \cos \beta_1 \frac{y}{b} \right) - k_1 \left( \sinh \beta_1 \frac{y}{b} - \sin \beta_1 \frac{y}{b} \right) \right] \cdot \Psi_{31} \\
M_{33}^y(x,y) = & -D \cdot \left\{ \left(\frac{\beta_3}{b}\right)^2 \left[ \left( \cosh \beta_3 \cdot \frac{y}{b} + \cos \beta_3 \frac{y}{b} \right) - k_3 \left( \sinh \beta_3 \frac{y}{b} + \sin \beta_3 \frac{y}{b} \right) \right] \cdot \right. \\
& \cdot \left[ \left( \cosh \beta_3 \cdot \frac{x}{a} - \cos \beta_3 \frac{x}{a} \right) - k_3 \left( \sinh \beta_3 \frac{x}{a} - \sin \beta_3 \frac{x}{a} \right) \right] - \\
& \left. -\nu \cdot \left(\frac{\beta_3}{a}\right)^2 \cdot \left[ \left( \cosh \beta_3 \cdot \frac{x}{a} + \cos \beta_3 \frac{x}{a} \right) - k_3 \left( \sinh \beta_3 \frac{x}{a} + \sin \beta_3 \frac{x}{a} \right) \right] \cdot \right. \\
& \left. \left[ \left( \cosh \beta_3 \cdot \frac{y}{b} - \cos \beta_3 \frac{y}{b} \right) - k_3 \left( \sinh \beta_3 \frac{y}{b} - \sin \beta_3 \frac{y}{b} \right) \right] \right\} \cdot \Psi_{33} \quad (13)
\end{aligned}$$

## RESULTS AND DISSCUSSIONS

Modal displacements values are given in tables 2 to 5. The total displacements (table 6). Bending moments for the modal responses to the axis x and y, are given (tables 7,8,9,10,11,12,13,14), and total bending moments (tables 15, 16).

*Displacements mode (1,3) Table 3.  $w_{13}$*

$b/a$	0	1	2	3
0	0	0	0	0
1	0	0,0019	0,0005	-0,0019
2	0	0,0035	0,0010	-0,0035

*Displacements mode (3,1) Table 4.  $w_{31}$*

$ba$	0	1	2	3
0	0	0	0	0
1	0	0,0058	0,0155	0,0200
2	0	-0,0060	-0,0159	-0,0205

*Displacements mode (3,3). Table 5  $w_{33}$*

$b/a$	0	1	2	3
0	0	0	0	0
1	0	0,001	0,0003	-0,0010
2	0	-0,001	-0,0003	0,0011

*Total Displacements) Table 6  $w_{tot}$*

$b/a$	0	1	2	3
0	0	0	0	0
1	0	0,0118	0,0247	0,0278
2	0	0,0022	0,0001	-0,0032

Bending moments mode (1,1) Table 7  $M_{11}^x$

b/a	0	1	2	3
0	0	-0,010	-0,027	-0,034
1	-0,0017	0,0006	0,0033	0,0045
2	-0,0031	0,0054	0,0176	0,0230

Bending moments mode (1,3) Table 8  $M_{13}^x$

b/a	0	1	2	3
0	0	0,0061	-0,0017	0,0062
1	-0,0018	0,0016	0,0005	0,0019
2	-0,0034	0,0055	0,0016	0,0062

Bending moments mode (3,1) Table 9  $M_{31}^x$

b/a	0	1	2	3
0	0	-0,0640	-0,1740	-0,2203
1	-0,0106	0,0454	0,1083	0,1293
2	-0,0196	-0,0351	-0,1186	-0,1706

Bending moments mode(3,3) Table 10  $M_{33}^x$

b/a	0	1	2	3
0	0	-0,0113	-0,0032	0,0115
1	-0,0010	0,0075	0,0022	-0,0078
2	0,0010	-0,0086	-0,0024	0,0089

Total bending moments Table 11  $M_{tot}^x$

b/a	0	1	2	3
0	0	-0,0916	-0,2034	-0,2374
1	-0,0152	0,0552	0,1142	0,1240
2	-0,0250	-0,0328	-0,1018	-0,1449

Bending moments mode(1,1)Table12  $M_{11}^y$

b/a	0	1	2	3
0	0	-0,0020	-0,0054	-0,0070
1	-0,0084	-0,0018	0,0036	0,0058
2	-0,0155	-0,0025	0,0090	0,0136

Bending moments mode (1,3)Table 13  $M_{13}^y$

b/a	0	1	2	3
0	0	0,0012	-0,0003	0,0012
1	0,0092	0,0050	0,0016	0,0067
2	0,0169	0,0097	0,0031	-0,0128

Bending moments mode (3,1) Table 14  $M_{31}^y$

b/a	0	1	2	3
0	0	-0,0128	-0,0343	-0,0441
1	-0,0156	0,0042	0,0271	0,0369
2	0,0160	-0,0053	-0,0303	-0,0411

Bending moments mode (3,3) Table 15  $M_{33}^y$

b/a	0	1	2	3
0	0	-0,0023	-0,0006	0,0023
1	-0,0050	0,0040	0,0012	-0,0050
2	0,0051	-0,0043	-0,0013	0,0053

Total bending moments. Table 16  $M_{tot}^y$

b/a	0	1	2	3
0	0	-0,0183	-0,0407	-0,0475
1	-0,0382	0,0115	0,0336	0,0310
2	-0,0112	-0,0023	-0,0195	-0,0350

Watching these modal displacements is observed that it obtain the maximum modal displacement corresponding to vibration mode (3,1). The maximum total displacements is obtained into the node (1,3). Regarding modal bending moments relative to the axis x is established that the maximum value is recorded in the node (0,3) for mode vibration (3,1). The maximum value of total bending moment relative to x axes is recorded in node (0,3)  $|M_x| = 0,2374$ . For modal bending moments relative to the axis y is established that the maximum value is recorded in the node (1,3) for mode vibration (3,1). The maximum value of total bending moment relative to y axes is recorded in node (0,3)  $|M_y| = 0,0475$ . It is observed that the value of bending moment in the x axis is 80 % higher than the bending moment in relation to the y axis. Following the modal efforts values relative to the x and y axes it can be draw the conclusions: For mode (1,1);  $|M_{11}^x| = 0,034$ , node (0,3);  $|M_{11}^y| = 0,0155$ , node (2,0)

Bending moment in the x axis is 55 % higher than the bending moment in relation to the y axis.. For mode (1,3);  $|M_{13}^x| = 0,0062$ , node (0,3);  $|M_{13}^y| = 0,0169$ , node (2,0). Bending moment in the y axis is ~ 64 % higher than the bending moment in relation to the x axis. For node (3,1);  $|M_{31}^x| = 0,2203$ , node (0,3) ;  $|M_{31}^y| = 0,0441$ , node (0,3). Bending moment in

the x axis is ~ 80 % higher than the bending moment in relation to the y axis. For node (3,3);  $|M_{33}^x| = 0,0115$ , node (0,3) ;  $|M_{33}^y| = 0,0053$ , node (2,3). Bending moment in the x axis is ~ 54 % higher than the bending moment in relation to the y axis.

## CONCLUSIONS

This paper presents the modal analysis that determines the total and modal dynamical responses in displacements and bending moments for rectangular plate loaded with a uniformly distributed forces over the entire surface. Using the multiplier dynamic function is simplifies the dynamic methodology calculation of plates. The method considered the shapes functions of plates vibrations as the product of bars shape functions having the same limits conditions. For symmetric normal modes was considered as known the own pulsations. By applying the superposition principle is determined the total dynamic responses in displacements and efforts. Also the paper presents the sectional efforts modal percentage deviations values between relative x and y axes. The results obtained in this paper reveal the most dangerous sections of the structure relative to the x and y axes. Having the necessary data will be able to perform the correct strength calculation.

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