THEORETICAL AND COMPARATIVE STUDY REGARDING THE MECHANICAL RESPONSE UNDER THE STATIC LOADING FOR DIFFERENTS RECTANGULAR PLATES

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Abstract

The mechanical response of rectangular plates is still a problem for the researchers in order to find and develop for the production the materials capable of taking over and transmitting the external forces to the structures in maximum safety conditions. Also, the production costs is an important factor that researchers should take into account in their analyzes.The paper represent a comparative and theoretical study about the approximate values of static displacements of the median surface points for the considered plates.

Key words: analitycal, method, plates, static, reinforced.

INTRODUCTION

In the present paper two types of plates were considered: the first is an rectangular plate made of wood refuse and the second is a rectangular "intelligent" reinforced plate with fiber glass. The analysis of the plates made from wood refuse was done using the variational Ritz method (Catarig., 2001) . The fiberglass reinforced plate study and analysis was made using the Ritz variational method. The both plates were considered to be simply supported on their contours and loaded by a uniformly distributed system forces on their median surfaces. Plates will only be represented by their median surface, (Fetea., 2010) because the mechanical displacements in any section is similar with the mechanical displacements of the median surface points.

The both plates have the same dimensions: length 100 [cm], width 100 [cm] and thickness 10 [cm].

$L = 100[cm];$

$l = 100[cm];$

$g = 10[cm]$

Where, $g = 10[cm]$, plate thickness.

For the wood refuse plate the elastic characteristics are the following:

- Young's modulus $E^{'} = 0{,}28 \cdot 10^5 \left[\frac{daN}{cm^2}\right];$
- the transverse contraction coefficient $\mu = 0$

- wood refuse density $\rho_1 = 0{,}6\left[\frac{g}{cm^3}\right]$

For the latice reinforced plate we have choose the following dimensionals and elstic characteristic:

- lattice density $\rho_2 = 0{,}9\left[\frac{g}{cm^3}\right]$;
- lattice thickness $g_2 = 0{,}5[cm]$;
- lattice hole have the dimension $2x2\left[cm^2\right]$;
- lattice length $a = 50[cm]$;
- lattice width $b = 50[cm]$;
- lattice volume $V_2 = 1250\left[cm^3\right]$;
- lattice mass $m_2 = \rho_2 \cdot V_2 = 1175[g]$;
- wood refuse volume $V_1 = L \cdot l \cdot g - V_2 = 98750[cm^3]$;
- wood refuse mass $m_1 = \rho_1 \cdot V_1 = 59250[g]$;
- „smart' plate volume $V = V_1 + V_2 = 100000[cm^3]$;
- „smart' plate mass $m = m_1 + m_2 = 60425[g]$.

The Young modulus for the wood refuse plate is

$$E = 0{,}28 \cdot 10^5\left[\frac{daN}{cm^2}\right]$$

MATERIAL AND METHOD

The Ritz method consist in selecting a suitable infinites series expression for displacements of the median surfaces points (Hadar A., 1998, Posea., 1991). As we know, these displacements must satisfy the geometrical boundary conditions, and the differential equation of movement is not required (Fetea M., 2010). To determine the points displacements in median surfaces of the plate was used the relation:

$$w(x, y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} X_i(x) Y_j(y) \tag{1}$$

- c_{ij}, unknown coefficients;
- $X_i(x)$, the approximate functions of displacements in the X directions of the plates;
- $Y_j(y)$, the approximate functions of displacements in the Y directions of the plates;

These functions $X_i(x)$, $Y_j(y)$, satisfy individual the geometrical boundary conditions (Ivan M., 1997). Median surface of plates is related to a system of triortogonal axes $OXYZ$.

OX and OY axes are contained in the median surface of the plate and the OZ axis is perpendicular to them having the positive direction downwards. The OX axis is confused with the length (L) of the plates and the OY axis with the width (l) of the plates.

The boundary conditions for the simply supported plates are (Munteanu, 1998):

$$w = 0, M_X = 0 \text{, for } x = 0, L;$$
$$w = 0, M_Y = 0 \text{, for } y = 0, l;$$

The variational Ritz method represent in fact the applying of minimal potential energy theory (Ciofoaia., 1986). For the simply supported plate, the shape function was chosen as the product of two trigonometric functions having x, y, variables (Posea N., 1976, Rosca I., 2002). So, we represents the displacements of median surfaces like a double infinites series:

$$X_i(x) = \sin\frac{m \cdot \pi \cdot x}{L}; \tag{2}$$

$$Y_j(y) = \sin\frac{n \cdot \pi \cdot y}{l}; \tag{3}$$

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} X_i(x) Y_j(y) = \sum_{i=1}^{m}\sum_{j=1}^{n} \sin\frac{m \cdot \pi \cdot x}{L} \cdot \sin\frac{n \cdot \pi \cdot y}{l} \tag{4}$$

Imposing the condition of minimum total potential energy was determinated the unknown coefficients (Martian, 1999, Missir-Vlad., 2002).

$$\Pi = 0$$
$$\frac{\partial \Pi}{\partial c_{ij}} = 0$$

Where, Π, represent the total potential energy.

It considered the assumption by which is negligible the intensity of weight forces of the glass fibre from the simply supported plate.

Regarding external force acting on the median surface of the plate, it is uniformly distributed with the intensity q.

Taking into account the rigidity and the loads value of forces, the points displacement in the median surface of the plate was determined with the relationship:

$$w(x,y) = \frac{16q}{\pi^4 D} \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\sin \frac{i\pi x}{a} \cdot \sin \frac{j\pi y}{b}}{\left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)} \tag{5}$$

D, bending rigidity of „smart” plate (Fetea M ., 2010);
w, displacement points from middle surface;
$i = 1, j = 1$.
Where,

$$D = \frac{Eg}{12(1-\mu^2)} \tag{6}$$

RESULTS AND DISCUSSION

For a correct calculation must be known proportions of fiberglass and wood mass of plates volume.

For the fibre glass

$$K_2 = \frac{V_2}{V} = 0{,}0125 .$$

For the wood refuse

$$K_1 = \frac{V_1}{V} = 0{,}9875$$

Having determined the proportion of components the „smart” plate density was calculated with the relation

$$\rho = K_1 \cdot \rho_1 + K_2 \cdot \rho_2 = 0{,}604 \left[\frac{g}{cm^3} \right]$$

The Young’s modulus for the wood refuse and fibre glass is:
a. for the wood refuse

$$E_1 = 0{,}1 \cdot 10^6 \left[\frac{daN}{cm^2} \right]$$

b. for the fibre glass

$$E_2 = 0{,}175 \cdot 10^5 \left[\frac{daN}{cm^2} \right]..$$

So, the „smart” plate Young’s modulus is

$$E = K_1 \cdot E_1 + K_2 \cdot E_2 = 0{,}989 \cdot 10^5 \left[\frac{daN}{cm^2}\right]$$

The wood refuse plate and „smart" plate rigidity are:

$$D_1 = \frac{E_{\cdot} g}{12(1-\mu^2)} = 0{,}2333 \cdot 10^4, \ \mu = 0$$

$$D_2 = \frac{E \ g}{12(1-\mu^2)} = 0{,}8241 \cdot 10^5$$

For the both plates the calculus for the maximum displacements was made in the middle of plates having the coordinates $x = 50[cm], y = 50[cm]$

The intensity of weight forces is determiated with the relation

$$q \cdot V = \rho \cdot V \cdot g$$

$$q = \rho \cdot g = 0{,}6 \cdot 10^3 = 600\left[\frac{N}{cm^2}\right]$$

Using the relation (5) the displacements in the middle of wood refuse plate is

$$w(x,y) = \frac{16q}{\pi^4 D}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\sin\frac{i\pi x}{a}\cdot\sin\frac{j\pi y}{b}}{\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)} = 0{,}25[cm],$$

For the „smart" plate

$$q = \rho \cdot g = 0{,}604 \cdot 10^3 = 604\left[\frac{N}{cm^2}\right]$$

$$w(x,y) = \frac{16q}{\pi^4 D}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\sin\frac{i\pi x}{a}\cdot\sin\frac{j\pi y}{b}}{\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)} = 0{,}072[cm]$$

CONCLUSIONS

Following the study using the Ritz method calculation were obtained in the center median plate surface displacements. $w(x,y) = 0{,}25[cm]$, for the wood refuse plate and $w(x,y) = 0{,}072[cm]$, for the „smart" plate

In a percentage analysis of the displacement values, it is found that the points displacement of the smart plate is only 28.8% from the displacement of the wood refuse plate.

It could be consider that these results regarding the mechanics displacements of the plates are consisted of a theoretical computational novelty which highlights the positive aspects regarding the takeover of the external loads.

As a conlusion, the „smart" plate can be use in the light construction fields because will reduce the weight forces of the construction, the price of building and can be introduced into production for a variety of dimensional characteristics depending on where they are deployed.

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