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FREE VIBRATIONS STUDY OF WOODEN RECTANGULAR PLATE HAVING A FREE EDGE

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Abstract

Initially, in the case of rectangular plate it was considered that is sufficient be taken into account only the resonance phenomenon. Later it being understood that vibration systems located away from the resonance can even modify and alter the structure of the materials. The literature reflected that the assessment respectively static dynamic response of flat plate is done taking into account the shape of the median surface, boundary conditions, loading mode and solving method adopted.

Key words: plate, free, vibrations, eigenfunctions, shape

INTRODUCTION

The problem of dynamic plate calculation is formulated taking into account Kirchhoff's simplifying assumptions. As is known, free undamped vibrations of rectangular flat plates is a problem of eigenvalues. Calculation of rectangular thin plate to solve the problem of free and forced vibration is reflected in the literature with contributions by several authors such as: (Warburton, 1954) which presented a set of solutions for the six cases of rectangular plates, (Janich, 1962) give a complete set of solutions for 18 of the 21 possible combinations of boundary conditions, (Iguchi, 1938), (Fletcher, 1959) whose concerns were directed to solve free vibration using semianalytic methods, respectively (Hamada, 1959) which has studied the plates by variational methods. Calculation programs for eigenfunctions, shapes plate, was designed by the author in Excel, and for frequency parameters in Matlab.

MATERIAL AND METHOD

Research on the present paper were performed in the installations for constructions research laboratories of the Technical University of Cluj-Napoca, Faculty of Building Services. The research and the determination of results was done between April 2017 - July 2017. The results of the studies and analyzes carried out falls into the fundamental theoretical research. Study methods applied are analytical calculation methods. The study conducted by the author can be framed in the fundamental theoretical studies. It is known that most of the complications that appear in flat plates solving are related to the existence of free sides (Posea, 1976; Fetea, 2009). These difficulties are reflected by the impossibility of finding functions to describe the stress in the plate having rigorous static conditions on free side (Barsan, 1971; Young, 1950). Scope of this paper is represented by the study of free vibration for rectangular plate clamped on two opposite sides and simply supported and free on the other two (SS-F-C-C). Leissa, Szilard, Fletcher, Timoshenko, were among the most important contributors to solving these types of rectangular plates. Flat plate analyzed is considered as being thin, elastic, isotropic with bending stiffness and meets the conditions of validity of Kirchhoff's hypothesis (Bors, 2005; Young, 1950). Is considered a rectangular plate presented in Fig. 1, with dimensions a and b clamped in the sides $x = 0, x = a$, simply supported on the side $y = 0$ and free on the side $y = b$.. The wooden plate is presented in Fig. 1.

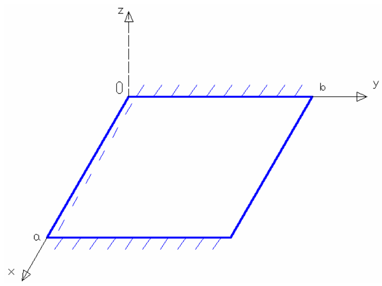

Fig. 1. Median surface of wooden plate

For the application of the variational method is considered on the surface of the plate a network consisting in 10 lines parallel to the axes OX and OY for determining in the nodes of the network the eigenfunctions values.**T**he median surface with the network designed in which nodes are determined the natural eigenfunctions values is presented in Fig. 2 (Fetea, 2009).

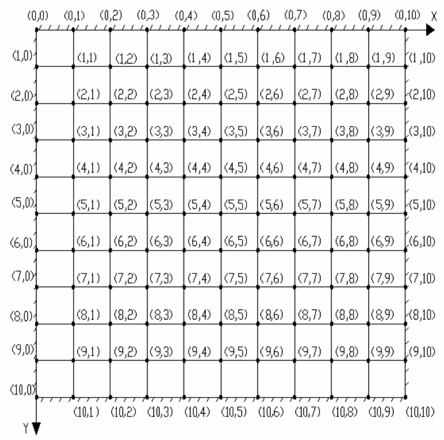

Fig. 2. Median surface network

In order to solve the problem of the own free vibrations of the plate by applying the variational method considered, the author went through the following steps:

1. The author present the eigenfunctios expression of beam.To determine the eigenvalues pulse parameters are used the expressions of beams shapes functions of vibration on the X and Y directions are (Leissa, 1969):

$$G_i(x) = \left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i\left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)$$

$$F_1(y) = \sqrt{3}\frac{y}{b},\ j = 1 \tag{1}$$

$$F_j(y) = \left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{y}{b}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right),\ j > 1$$

The parameter values $\beta_i, k_i,\ \beta_j, k_j$, were determined in (Fetea, 2009). The expressions of shapes functions (Fetea, 2009; Leissa, 1969) for the rectangular plate is represented like the product of beams eigenfunctions:

$$\Phi_{ij}(x, y) = G_i(x) \cdot F_j(y) \tag{2}$$

2. In the second stage of work the author presents the problem of eigenvalues of Sturm - Liouville. The problem of own values associated with this plate, can be written in the form:

$$\nabla^4\Phi_{ij}(x, y) = \lambda_{ij}\Phi_{ij}(x, y) \tag{3}$$

3. The third step in solving the rectangular plate consists in replacing the eigenfunctions of plate given by the product of beam eigenfunctions in the expression (2) and integrating on the whole field of the plate by applying the method of separating the variables.

4. In the fourth stage of plate solving, the author determined the expressions of pulse parameters for the first 3 normal modes of vibration.

5. Using a numerical computation program, the author determined the values of the plate eigenfunctions of his own vibrational forms of the slab in the nodes network considered in Fig. 2.

6. In the last step using the calculated numerical program, the author presents the deformed shapes of the plate for the first three normal modes of vibration.

RESULTS AND DISSCUSION

Starting from the equation:

$$\nabla^4\Phi_{ij}(x, y) = \lambda_{ij}\Phi_{ij}(x, y) \tag{4}$$

taking into account the functions of the beams the author adopt the notations:

$$I_1 = \left(\frac{\beta_i}{a}\right)^4 \int_0^a [(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}) - k_1(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a})]^{IV}$$

$$\cdot\left[\left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i\left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right]dx$$

$$I_2 = \int_0^b G_i^2(x)dx = \int_0^a \left[\left(\cosh\beta_i \cdot \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i\left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right]^2 dx$$

$$I_3 = \int_0^b F_j^2(y)dy = \int_0^b \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{y}{b}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]^2 dy,$$

$$I_4 = \left(\frac{\beta_i}{a}\right)^2 \int_0^a G_i''(x) \cdot G_i(x)dx =$$

$$\left(\frac{\beta_i}{a}\right)^2 \int_0^a \left[\left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i\left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right]'' \cdot$$

$$\cdot\left[\left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i\left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right]dx,$$

$$I_5 = \left(\frac{\beta_j}{b}\right)^2 \int_0^b \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]'' \cdot$$

$$\cdot\left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]dy,$$

$$I_6 = \left(\frac{\beta_j}{b}\right)^4 \int_0^b \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]^{IV} \cdot$$

$$\cdot\left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j\left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]dy \cdot$$

Integral values determined for $G_i(x)$, $F_j(y)$, are known and are presented in. For the square plate corresponding to the normal modes of vibration (1,1), (2,1), (3,1) is obtained pulsation parameter values, shown in Table 1.

Table 1

Pulsations parameters of plate

Modes	(1,1)	(2,1)	(3,1)
$\sqrt{\lambda_{ij}}$	22,37	61,6	102,9

The values of the natural vibration shapes eigenfunctions are shown in Tables 2-4.

Table 2

Shapes functions for mode (1,1)

		Vibration Functions							Mod (1,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,032754	0,065507	0,098261	0,131015	0,163769	0,196522	0,229276	0,26203	0,294783	0,327537
0,8	0	0,107282	0,214564	0,321846	0,429128	0,536409	0,643691	0,750973	0,858255	0,965537	1,072819
1,2	0	0,189833	0,379666	0,569498	0,759331	0,949164	1,138997	1,32883	1,518662	1,708495	1,898328
1,6	0	0,252091	0,504183	0,756274	1,008366	1,260457	1,512549	1,76464	2,016731	2,268823	2,520914
2	0	0,275075	0,55015	0,825226	1,100301	1,375376	1,650451	1,925526	2,200601	2,475677	2,750752
2,4	0	0,252092	0,504183	0,756275	1,008367	1,260458	1,51255	1,764641	2,016733	2,268825	2,520916
2,8	0	0,189833	0,379666	0,5695	0,759333	0,949166	1,138999	1,328833	1,518666	1,708499	1,898332
3,2	0	0,107283	0,214565	0,321848	0,429131	0,536413	0,643696	0,750979	0,858261	0,965544	1,072827
3,6	0	0,032755	0,06551	0,098265	0,13102	0,163775	0,19653	0,229285	0,26204	0,294795	0,32755
4	0	2,14E-06	4,28E-06	6,43E-06	8,57E-06	1,07E-05	1,29E-05	1,5E-05	1,71E-05	1,93E-05	2,14E-05

Table 3

Shapes functions for mode (2,1)

		Vibration Functions							Mod (2,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,144501	0,265661	0,343479	0,364099	0,321605	0,218483	0,064626	-0,12508	-0,33436	-0,5499
0,8	0	0,473301	0,870148	1,125034	1,192575	1,053389	0,715622	0,211675	-0,4097	-1,09518	-1,8014
1,2	0	0,837495	1,539706	1,990721	2,110234	1,863947	1,266276	0,374555	-0,72495	-1,93789	-3,1875
1,6	0	1,112165	2,044677	2,643609	2,802318	2,475257	1,681572	0,497396	-0,96271	-2,57345	-4,2330
2	0	1,213563	2,231094	2,884633	3,057811	2,700932	1,834884	0,542745	-1,05048	-2,80808	-4,6189
2,4	0	1,112165	2,044678	2,643611	2,80232	2,475259	1,681573	0,497396	-0,96271	-2,57345	-4,2330
2,8	0	0,837497	1,539709	1,990725	2,110239	1,863951	1,266279	0,374556	-0,72495	-1,93789	-3,1876
3,2	0	0,473304	0,870154	1,125042	1,192584	1,053396	0,715627	0,211677	-0,4097	-1,09518	-1,8014
3,6	0	0,144507	0,265671	0,343492	0,364114	0,321618	0,218492	0,064628	-0,12509	-0,33438	-0,5500
4	0	9,45E-06	1,74E-05	2,25E-05	2,38E-05	2,1E-05	1,43E-05	4,23E-06	-8,2E-06	-2,2E-05	-3,6E-05

Table 4

Shapes functions for mode (3,1)

		Vibration Functions							Mode (3,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,348247	0,64024	0,827781	0,877476	0,775065	0,526542	0,155747	-0,30145	-0,80581	-1,32546
0,8	0	0,92212	1,695286	2,191874	2,323463	2,05229	1,394228	0,412402	-0,7982	-2,1337	-3,50969
1,2	0	1,150413	2,114994	2,734524	2,898691	2,560383	1,739402	0,514502	-0,99582	-2,66195	-4,37859
1,6	0	0,790552	1,453403	1,879138	1,991952	1,75947	1,1953	0,35356	-0,68432	-1,82927	-3,00892
2	0	6E-06	1,1E-05	1,43E-05	1,51E-05	1,34E-05	9,08E-06	2,68E-06	-5,2E-06	-1,4E-05	-2,3E-05
2,4	0	-0,79054	-1,45337	-1,8791	-1,99191	-1,75943	-1,19528	-0,35355	0,684301	1,82923	3,008864
2,8	0	-1,15038	-2,11494	-2,73445	-2,89861	-2,56032	-1,73936	-0,51449	0,99579	2,661881	4,378476
3,2	0	-0,92206	-1,69517	-2,19172	-2,3233	-2,05215	-1,39413	-0,41237	0,798147	2,133554	3,509441
3,6	0	-0,34811	-0,63998	-0,82745	-0,87712	-0,77475	-0,52633	-0,15568	0,301327	0,805488	1,324931
4	0	0,000307	0,000563	0,000729	0,000772	0,000682	0,000463	0,000137	-0,00027	-0,00071	-0,00117

The deformed forms corresponding to the considered normal modes of vibration are shown in Fig. 3-5.

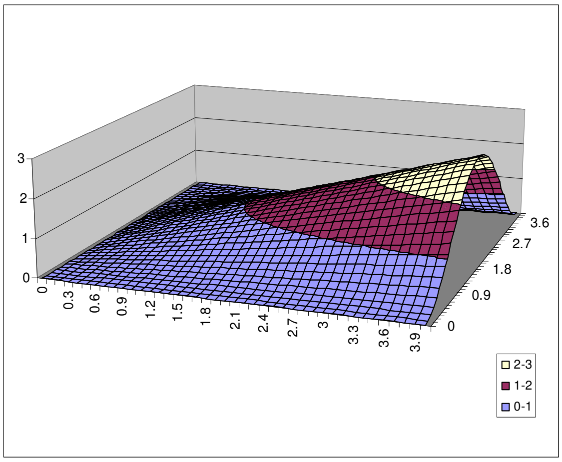

Fig. 3. Image of shape corresponding to mode (1,1)

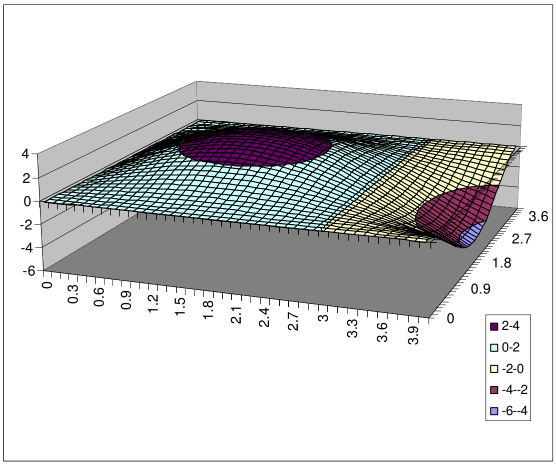

Fig. 4. Image of shape corresponding to mode (2,1)

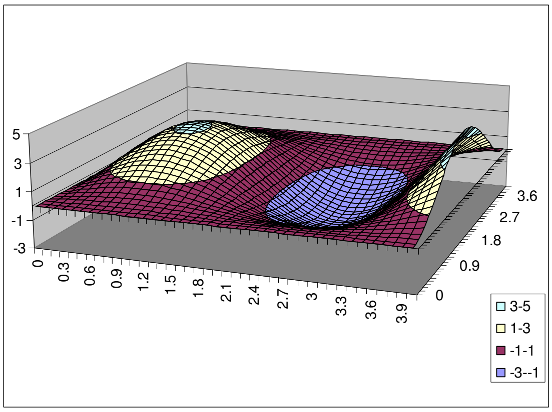

Fig. 5. Image of shape corresponding to mode (3,1)

CONCLUSIONS

Solution of such problems by the classical methods is very difficult to find, or often impossible. The Vlasov's method used by the author in this paper, uses linear combinations of the eigenfunctions of lateral beams vibrations, which are able to satisfy most boundary conditions. In the case of Galerkin-Vlasov variational method adapted by the author are highlighted as follows:

- The election of displacement function approximation as a linear combination between the natural vibration shapes products functions of the beams on both directions and time function which indicates that plate motion after a normal harmonic vibration which is a motion that is produce a specific pulsation;
- Calculation algorithm, which characterizes the method;
- Pulsations parameters considered for normal modes of vibration;
- The shapes funcions for three normal modes of vibration.

In the literatureare are not published results regarding normal modes of vibration of this type of plate, the only references that can be considered are those presented by Barsan, Fletcher and Leissa, which states that for the case of antisymmetric-antisymmetric vibrational mode (2.1), the parameter values are close to those determined for the case of plate clamped on two opposite sides and free on the other two. Also, an approximate value of their fundamental parameter for the square plate is given by Janich . Using the Rayleigh method, (Janich, 1962) considered shapes functions of beams vibrations as given by simple trigonometric functions. By applying the Rayleigh method, Janich obtained for the fundamental pulse parameter the value [12] $\sqrt{\lambda_{11}} = 24{,}64$. Considering the approximate percentage deviation of the parameter values determined by the method proposed, the fundamental parameter, compared to parameter value determined by Janich [12], is 9.22%. The paper proposed by the author is intended to be an attempt to validate the Vlasov-Galerkin variational method for flat plate considered. The paper includes not only information from the Romanian and international literature of teachers and researchers dynamics schools, but also the values determined by the author on the characteristic dynamic determined. For the first 2 normal vibration modes, Barsan obtained: $\lambda_{11} = 12{,}757$, $\lambda_{21} = 63{,}212$.
and Fletcher obtained using other methods the following values for the pulsation parameters: $\lambda_{11} = 12{,}69$, $\lambda_{21} = 63{,}01$.

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