A REVIEW OF TREND METHODS FOR CALCULATING THE TREE VOLUME. TRENDS AND NEW APPROACHES

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Abstract

The volume of the trees can be approximated by evaluating their taper form, which represents a mathematical modeling of some geometric solids. Since the first methods of calculating the volume to current formulas there were used methods of assimilating the taper form of the tree trunk with various geometric solids whose volumes were known. With the development of mathematics and computer engineering it becomes increasingly obvious that the tree volume can be calculated more easily and with greater accuracy using mathematical equations. The proposed equations are equations that generates the profile of tree trunk. Thus, as the profile curve generated by the equation is closer to the natural model, so the results in terms of volume calculations are more accurate. This article presents the evolution of concepts and methods used to calculate the volume. It is also defined the concept of taper profile and taper form equations with applications in the calculation of trees volume.

Key words: review, taper form, mathematical models

INTRODUCTION

The first studies on the tree volume began in the nineteenth century. Around 1804 Heinrich Cotta first introduced the notion of yield table (Clark, 1991). A large study in this respect being accomplished in Norway for spruce species.

The taper form was studied after calculating the volume, and it consisted in introduction of mathematical models that can describe the decreasing diameter along the spindle shaft. Later, these equations were used to calculate the volume, some authors considering that the taper form equations are more accurate than volume equations.

In the last 100 years there have been developed lots of equations that reproduce the taper form, for a high variability of forms. So it started from tapered simple functions (Behre, 1923; Hilt, 1980) then the complex ones, calculated segmented (Max and Burkhart, 1976; Demaerschalk and Kozak 1977; Cao et al., 1980; Clark et al., 1991). Most of these equations are applied to a single species or group of species. For example, model Max and Burkhart (1976) was developed for pine; Hilt model has been developed for oak (Quercus spp.), etc.

Even if more than 100 years there are many studies for both types of equations, for volume or for taper form, yet there is no unique model that can be used with good results for all species (Muhairwe, 1999). In 1970, Mitscherlich notes that taper form variability is a result of differences in the rate of growth of diameter at different heights.

Muhairwe researches conducted in 1994 underline the fact that the variability of taper form is determined by several factors, the variation of species, climate, site quality, age of the trees and stand, defoliation, density. Throughout the last century the taper form has been carefully studied, and there were developed many models of regression equations which can reflect accurately the taper form of trees.

The progress of understanding tree taper form

Taper form means the characteristic of the tree trunk from the ground level to the top with respect to the relation between diameter and height (Burkhart and Tome, 2012). The taper form of a tree may also be defined as the manner in which the diameter changes its value in relation to the height along the trunk. There are several ways to describe the taper form. The taper form can be expressed as a curved spindle, as a profile of the trunk, in a relation/report (indices, coefficients).

The taper form equations can be divided into several categories depending on their form. The development of mathematical models that reflect taper form is based on the same division of the taper into geometric solids which can capture as realistic the natural form. Starting from the spindle taper, mathematical models are composed of sub-functions that best reflect each geometric shape. These sub-functions of the mathematical model join at specific points on the taper like base of the crown or a relative predetermined height (Max and Burkhart, 1976; Valentine and Gregoire, 2001).

A first classification is performed by Methol, 2001, who is grouping the taper form equations into four categories, namely: simple equations; polynomial segment equations; exponential equations respectively mixed equations. The latest classification belongs to the researchers Dieguez-Aranda et al., 2006, and Burkhart and Tome, 2012, which are dividing the mathematical models that describe the taper form into three groups.

Simple mathematical models (simple equation) attempts to define a simple mathematical model as a polynomial equation belongs to Matte 1949: The mathematical model is built on the fact that the exponential power value varies depending on the geometrical shape of the taper segments. West, 2004 has determined the values of the constant r as -1.5 for the taper base, neiloid assimilated form; 0.5 for the middle segment of taper, paraboloid assimilated shape and one for the upper taper (which has the most pronounced decreasing to the top) assimilated to geometric cone.

Based on the general form of the above equation, various mathematical models have been developed, describing the taper form with various transformations (Ormerod, 1973). Many of these models calculated on taper segments have proven difficult to integrate into a single model to calculate the volume (Thomas and Parresol, 1991). The mathematical model proposed eliminates the taper segmentation into geometric solids by treating the taper shape as a continuous function, (West, 2004). In 1968, Bruce et al., propose a mathematical model based on the quadratic equation. Low degree polynomials have the disadvantage that do not accurately describe the lower portion of the trunk where we have a sprawl. Kozak et al. (1969) propose a parabolic function that adjusts the calculation of taper form.

Research has revealed that this quadratic equation gives good results for the middle part of the taper but fails to adequately describe the top of the taper and basal trunk sprawl. To correct the errors that occur in the description of the upper part of the taper, the equation can be calibrated so that when h_i is equal to h, estimated diameter to be zero. To achieve this it requires that $b_0 = - b_1 - b_2$, thus resulting a simplified equation as a regression equation with two parameters without constant term.

Researches performed to find mathematical models that describe best the taper form have not stopped at the quadratic equation, so that it began to be used higher order polynomial (Bruce et al. 1968; Hilt, 1980). Hilt (1980) improves the model proposed by Bruce et al., (1968) using a polynomial of higher order. Laasasenaho (1982) proposes another function as a higher order polynomial according to a Fibonacci number of exponents.

The model used to generate the taper form to the base diameter located higher height, excludes the tree base from the mathematical model because as Bruce (1972) noted, excluding this part of the tree we get a greater accuracy and simpler models. Rustagi and Loveless, 1991 developed a full simple function that allows both volume calculation and determination of dendrometrical elements height and diameter along the taper to individual trees.

The equation was developed by dividing the total height diameter and height measured at the base or at other heights (1/2, 2/3, 3/4). This started from Pressler theory, 1864 taken over by Loetsch et al. 1973, that divides taper into two parts, the part above the height corresponding to the base diameter (BH), and the portion between the ground and the height corresponding to the base diameter. Volume for the section between the ground and the appropriate base diameter height is approximated as a cylinder.

Mathematical models calculated by segments is second models describe the taper form in different sections of trees, by using different forms and equations. The taper division into several parts, representing geometric base forms is not a new idea. Since 1837 Smalian divided the taper into truncated cones with different skills exponents (Bruce and Max, 1990). The novelty is the development of equations describing continuous curves and continue derivatives and by using the integrals of these functions they estimate the volume of the section or of the taper.

Although the taper form of trees cannot be fully described using mathematical models, however, it is accepted that certain portions of the taper to resemble various geometric solid. Thus, the lower portion of the trunk is assigned to the neiloid trunk, middle portion to the trunk paraboloid, and the upper portion to the cone.

From this statement, we can consider that to describe the whole taper we need three functions: one for the base of the trunk, one for middle and one function for the top trunk. These three functions can form one single mathematical model using techniques of mathematical regression on mentioned segments. A first model was proposed by Max and Burkhart (1976), using the quadratic proposed by Kozak et al., (1969) to describe the taper form into three segments, using regression equations polynomial that divided taper into models to describe its conical shape. The mathematical models proposed in the form of polynomial equations calculated by segments consist of sub-models actually defined on each section of the taper. These sub-models that are joined together can form the polynomial equation model on taper segments.

Several authors recognized that other geometric shapes can be used to generate the taper form. The taper form of trees does not change suddenly from one geometric form to another; it is continuous, as shown Kozak (1988) and Newnham (1988). The introduction of computers in forest research in the early 1960s and the increased availability of the necessary software, together with the failure of simple mathematical models to track multiple inflection points along the trunk or the inability to match trunk segments together led to the development of more complex functions.

Mathematical models of equations based on segments describe each section of a tree with separate equations. The method commonly used to describe these forms is to match each section with a simple equation or a polynomial one, usually square, and then to generate a mathematical model for generating a continuous curve in the nearest points of the segments (Max and Burkhart, 1976; Demaerschalk and Kozak, 1977; Cao et al., 1980; Ormerod, 1986).

Various studies have shown that equations calculated on the segments, are better suited to the taper profile than simple models, in particular where the base of the tree is characterized by a sprawl (Cao et al., 1980; Walters and Hann, 1986).

After Heijbel 1928, according to Larson, in 1963, proposed a model cone formed by three sub-models, one for the base, one for the middle and one for the above, several other models in this class have been developed (Ormerod, 1986; Max and Burkhart, 1976; Demaeschalk and Kozak, 1977; Cao et al., 1980; Walters and Hann, 1986). Ormerod in 1973 was the first to develop a taper form equation with two sections. He started with a simple model from the model used by Bruce et al., 1968.

Due to changes in taper form on the trunk length, the simple equation above cannot provide an adequate description of taper form, therefore Ormerod proposes to amend equation for a step function and to fit separate exponents for each step. Ormerod is basing his mathematical model on the assumption that all trees have a single point of inflection at 30% of the total height. Max and Burkhart (1976) described the conic curve using segmented polynomial regression models based on statistical methods Fuller (1969) and Gallant and Fuller (1973). For strain, he assigned a different sub-polynomial model, which was defined in each section of the partition. By connecting sub-models at common points, he obtained a segmented polynomial model.

Mathematical models with variables - exponential equations, represent a continuous function that describes the taper form from the bottom to the top by using different exponents to model the shape of the taper on the three main segments namely: basic (neiloid), medium (paraboloid) and top (tip) which has a conical shape. In 1985 it is reiterated the idea of a single model to describe the form of the taper.

The first models were based on the models of Omerorod (1973) which have been applied by Reed and Byrne (1985) using as data the base diameter and total height. Applying this function showed that basal taper diameters are underestimated and the top diameters of the taper are overestimated (Muhairwe, 1993). Pioneering mathematical models to express the taper form by this process have been proposed by: Newnham, (1988) and Kozak (1988), also compatible models have been developed by: Perez et al., 1990; Newnham, 1992; Muhairwe et al., 1994; Bi, 2000; Westfall and Scott 2010. The first two equations with good results are those proposed by Newnham (1988) respectively Kozak (1988). Newnham (1988) offers a mathematical equation based on the model proposed by Ormerod (1973) with fixed values for p variable, for each section of the taper, so taper geometric model to vary continuously along it. Demaerschalk and Kozak (1977) indicated that the relationship between *di/d* și *hi/h* changes from neiloid the paraboloid at a fixed point (p) of the total height of the taper, called the point of inflection. As a result of their research it has shown that the inflection point is ranging from 20% to 25% of the total height from the ground. Besides, the relative height of the point of inflection has been fairly constant within a species, regardless of the size of the tree.

However, it should be noted that nowadays numerical techniques to obtain volume depending on the desired tree height are advanced, and using computer techniques combined with statistics and mathematics, lead to results that provide satisfactory precision. A current problem is trying to interpret highly complex mathematical functions of biological components that depend on a huge number of variables, most unknown. That is to emphasize the use of Bayesian statistics that express results in relation to the degrees of freedom. Such interpretations are just one of the ways of interpretation of probability, although there are other techniques that do not rely on the number of degrees of freedom.

After the pioneering work of Newnham (1988, 1992) and Kozak (1988, 1997) to obtain tapered variable exponent function, others have followed the same line of thought but using different equations and / or representatives conical base. Sharma and Zhang (2004) developed a variable exponent taper equation for black pine, spruce, fir trees grown in eastern Canada.

CONCLUSIONS

The advantage of forming mathematical models for calculating the volume derived from mathematical models of taper form have been highlighted by many authors, who noted that the possibility of calculating the total amount or at a specific height. So, the authors show at the same time the limited potential of a regression equation used to calculate the volume.

Also, recent studies show that measuring an additional diameter on the taper length increases the accuracy of estimation of the volume. It should be considered that not all mathematical models developed to predict the taper form can be integrated to calculate the volume. When integration is possible, mathematical models rendering the taper form are categorized as compatible models.

An equation is compatible when by integration it is obtain a taper volume identical to that obtained by using classical formulas - Huber or Smalian. An equation is not compatible when it can not be integrated in order to allow the calculation of the volume or by integration the volume obtained is erroneous. The future of volume estimating models is represented by neural networks.

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