WEIBULL MODEL FOR FREQUENCY AND CUMULATIVE DISTRIBUTION FUNCTION

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Abstract

In the present paper the authors evaluated the manner in which Weibull model is used for wind energy analysis. There are presented the parameters estimations for a mathematical model of wind turbine. In wind energy performance is very important to find the optimal model to construct certain methods and strategies for aeroelectrical aggregates dynamic. In order to do this we will use the Weibull probability density functions which are used to establish wind speed distributions in the case of wind energy applications. This study highlights the importance of certain parameters of wind turbine, in the process of construction and development of such aero-electrical aggregates and their optimal functioning. The frequency and cumulative distribution curves are plotted.

Key words: Wind turbines, Weibull distribution

INTRODUCTION

The main component that ensures the conversion of kinetic energy of wind into mechanical energy useable to turbine shaft, through the interaction between air current and moving blade, in the aero-electrical aggregates background, is the wind turbine. Wind energy is a renewable energy, ecological and sustainable. The optimization process of the construction of wind turbines is a continuing concern for researchers and manufacturers in the wind power field, having as finally purposes, solutions capable of performing a maximize economic efficiency of these aggregates.

Wind turbine is composed mainly of a rotator fixed on a support shaft, comprising a hub and a moving blade consisting of one or more blades. Active body of aeolian turbines which made the quantity of converted energy is the blade. The achieving of aerodynamic performances, kinematics and energy curves of the aeolian turbines depend on the choice of certain geometry.

The performance of the wind turbine can be investigated through mathematical models and also verified by experimental measurements.

The present study aims to present scientific knowledge in the field of wind turbines in order to select a mathematical model useful in aero-electric aggregates dynamic. It will be analyzed the Weibull statistical model for certain indicators on the wind speed distribution. All quantities of energetic interest depend on time, being statistically processed in form of frequency curve or insurance curve.

MATERIAL AND METHOD

The physical and mathematical model associated in aeroelectrical aggregates dynamic implies the following components: offer site (emplacement), sharing energy at the wind engine level, optimized geometry of the engine, characteristic curves.

Energy performance representation that produces a wind turbine, as a whole operating area, is materialized by the characteristic curves that are operating in the optimization process (Dubau, 2005).

Taking into consideration the paper (Dubau, 2003, 2004), in order to characterize the functionality of various types of turbines three adimensional coefficients are used, respectively: power coefficient, moment coefficient or torque coefficients and axial force coefficient. All sizes with energy interest are time-dependent statistically being statistically processed on the form of frequency or insurance curves.

The analysis of efficiency of wind energy recovery technology aims to maximize the energy production at a time interval. The recovered energy depends on offer site (emplacement) and on performance of engine lines. The line of engine must be adapted at natural conditions given by sites or locations. Therefore, the valorization system should include also an evaluation model of the offer site (emplacement).

Usually the analyzed energy is made for quasi-stationary regime. In the context of a dynamical study it cannot be neglected the time variation of kinematic measurements. Kinematic power (P_c) associated with an exposed area of the recovery system is

$$P_c = \rho \frac{v^3}{2} S$$

where we have used the following notations:

ρ – Mass density of air [kg/m^3],

v – Wind speed [m/s].

We have to mention some useful considerations regarding the wind regime. Terrestrial boundary layer has an extension of hundreds of meters. We understand by terrestrial boundary layer that area which is influenced by the current friction with the solid boundary of the terrestrial surface. The wind profile within the boundary layer depends on the solid boundary roughness and the land geometry. The wind turbine is usually located within the boundary layer. In this sense the exposed are of the turbine have different speed. One uses the value of the speed at the turbine shaft when we refer to the energetically modeling. This represents an approximation and it is accepted for some models.

The wind regime it is not stationary. The speed vector depends on time.

The turbulence is a random variation of the speed in quantity and direction. Many aerodynamic phenomena depend on turbulence degree. At the very small speed the flow can be laminar. The turbulence has a wide spectrum of frequencies.

The gusts are turbulences within the field of speed caused by whirlwind which are generated by the terrestrial surface or by other turbulences. These whirlwinds are periodic movements characterized by determined frequencies. The turbine receives the gusts as an aerodynamic shock.

Regarding atmospheric stability we note that the heating of the atmosphere lower layers by contact with the ground can generate upward movements which disrupt horizontal movement of wind.

There are various types of wind turbines. Between various types of wind turbines the rapid axial horizontal wind turbines are the most development ones. Many studies are also elaborated taking in consideration the turbines with vertical axes. Such a study was presented in a recent paper (Dubau, 2009). A study regarding a vertical turbine V250 can be seen in the paper (Gyulai et al., 2000, Gyulai, Bej, 2000 and Gyulai, 2000 a.) and for the horizontal turbine H2500 we can consult the paper (Dubau, 2007). In this context we can also recall the paper (Bej, 2003).

In the present paper we propose a mathematical model which allows assessment of horizontal speed averaged in a time of order by minutes. The above mentioned model uses the box method associated with probable frequencies within a wide range of time interval of years order. The box is represented by a velocity interval with a central value and its correspondent interval.

This model identifies statistically the probable frequency distribution of the speed box. The frequency is associated with central value of the boxes. This model does not include the wide frequency spectrum of the turbulence, neither of gusts or atmospheric stability.

The proposed model is available for an average year within a multiannual interval. The shorter intervals (month, day and year) are sometimes used as fractions of years. These represent "average month", "average day" of a year.

The mathematical model use two types of curves namely: frequency distribution curve and cumulative or insurance distribution curve. These curves are related by derivative-integrating operations. The model is based on measurements made by meteorological stations. This model can be analytically approximated by Weibull functions.

This paper provides a mathematical simulation on the computer of Weibull approximation, based on the model proposed in the work (Dubau, 2006 and Dubău, 2007).

For certain values of the wind speed [m/s], the Weibull functions $FF(v)$ and $FA(v)$ was theoretical determined.

Using MAPLE commands we plot the frequency distribution curve and cumulative or insurance distribution curve.

By making use of the above mentioned mathematical software, as an advanced concept for engineering modeling activities, we provide a theoretical modeling which will be very useful for future practical aspects in designed technology.

RESULTS AND DISCUSIONS

In this section we will present the analytical approximation by Weibull functions in order to construct a mathematical model based on the certain speed domains characteristics for wind energy turbines (see also Burton, 2001).

The Weibull model for frequency distribution curve of the velocity is concretized on frequency distribution function

$$FF(v) = \frac{8760}{c} \cdot k \cdot \left(\frac{v-a}{c}\right)^{k-1} \cdot \exp\left[-\left(\frac{v-a}{c}\right)^{k}\right]$$

and the Weibull model for insurance curve is concretized on cumulative distribution function namely

$$FA(v) = 8760 \cdot \exp\left[-\left(\frac{v-a}{c}\right)^{k}\right].$$

The frequency consists on hours/year or by reference to those 8760 hours/year expressed in relative frequencies (adimensional). The Weibull curves consist on three constants:

k – form parameter;

c – scale parameter;

a – location parameter.

These parameters can be evaluated as functions of multiannual average velocity of the offer site (emplacement). Location parameter depends on the frequency which is associated with null speed $(v=0)$. In order to identify the parameters above mentioned, within this model we have the following relations.

Form parameter:

$k = K\sqrt{v_m}$, $K = 1.05,...,0.73$ (the average value is 0.94).

For offer site (emplacement) from USA, the used values by NASA are:

$K : 0.9144 \div 0.9002$ (the domain for $v_m = 5.58 \div 9.81\, m/s$).

Scale parameter:

$$c = \frac{v_m}{-0.09562 - 0.1236 \cdot k + 0.68605 \cdot \sqrt{k} + \dfrac{0.51928}{k}}, \ k = 1...7 .$$

Location parameter:

$$a = -c \cdot \left[\ln \frac{8760}{T_0} \right]^{1/k}$$

$$T_0 = 8760 - 3050 \cdot (v_m)^{-1.65}, \ \ \Delta T_c = 3050 \cdot (v_m)^{-1.65} \text{ [hours/year]}.$$

For this model the Weibull constants have the following values for different average speeds, pointed in the table below (Table 1):

Table 1

The values of Weibull constants for certain average speeds

$v_m\,[m/s]$	2	4	6	8	10
K=1.05					
k	1.48	2.10	2.57	2.97	3.32
c	2.29	4.51	6.75	9.00	11.25
a	-0.52	-0.92	-1.42	-1.99	-2.61
K=0.94					
k	1,41	1.88	2.30	2.66	2.97
c	2.17	4.50	6.67	8.89	11.12
a	-0.43	-0.76	-1.19	1.65	-2.17

The average speed is influences by the elevation of the turbine.

For certain values of the parameters which occurs in this model namely $k = 0.94 \cdot \sqrt{5} = 2.1$, $C = 5.642$, $a = -0.97$, we plot below the frequency distribution curve and insurance distribution curve of the turbines (Fig.1 and Fig.2).

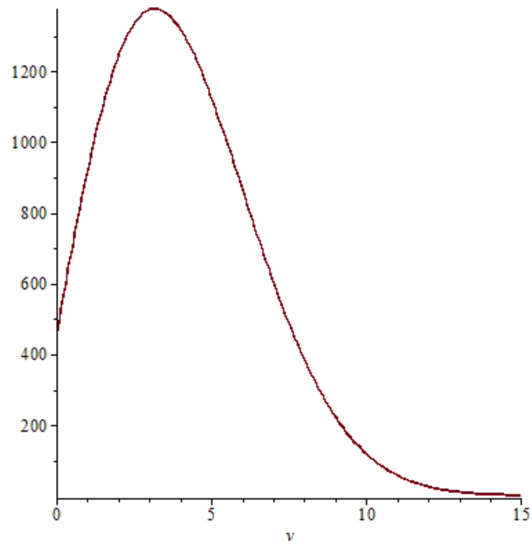

Fig. 1. Frequency distribution curve

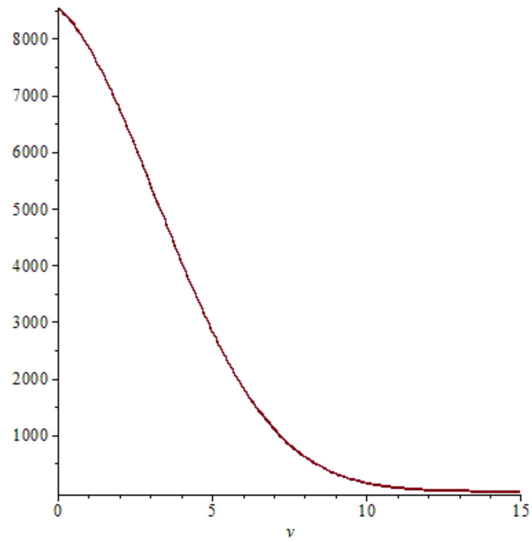

Fig. 2. Cumulative distribution curve

CONCLUSIONS

At any modeling process with energy objective it should be taken into consideration the offer site.

The frequency curve of the wind speed influences the designed technology characteristic which has to be centered on the speed domain. It should be choose that technology which has the following values of the speed emplacement:

- average speed or that of maximum frequency implies the place of the maximum efficiency of technology;
- high wind speeds with insurance of few hours per year those which define the powers and speeds installation technology;
- speeds installation can be two or three times the value of average speed;
- high accidental speeds requires protection measures of recovery technology.

The presented model for the analytical approximation permits a better optimization of the parameters of the turbine also the optimization of energy performance of the wind turbines. They are also permitted a good management of the control and automation. Significant differences occur only in the small rapidity domain.

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