REGRESSION EQUATIONS FOR VOLUME CALCULATION AT SPRUCE

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Abstract

Calculating the volume of trees can be achieved through mathematical equations. These equations are using as customary variables the diameter at breast height and the height of the tree. The mathematical model most used to calculate the volume in Romania is the double logarithmic equation (Giurgiu 1979), which underlies the method of yield tables. This material presents several mathematical models for calculating the volume. We will establish the coefficient of determination (R^2) and the error (s_e) for each model, following to choose the most appropriate equation to calculate the volume of wood for spruce species from a forest area with a restricted range. We will also determine the importance of diameter and height in mathematical models to calculate the volume. An important part in volume calculation is the curve of heights, in this sense we will analyze a series of mathematical models, we will establish correlative link between the diameter at breast height and height, in order to determine the optimum regression equation to generate the curve of tree form. This study was accomplished in a natural spruce stand aged 40 years, located in Vatra Dornei aria, Suceava county, Romania at an altitude of 1250 m.

Key words: volume, diameter, height, regression equation, correlation

INTRODUCTION

Given the requirements increasingly high and the constant pressure on the wood market, in the context of limited resources and the globalization of wood and wood products trade, the most precise calculation of wood volume is becoming increasingly important for players in the wood market. For economical reasons, for over two centuries, since dendrometrical measurements are made, the interest and concern for the scientific knowledge of tree volume, particularly of tree form were predominant (Giurgiu, 2004). To calculate the volume as accurate as possible, yield tables and regression equations were developed. Giurgiu defines the yield table for trees as a range of values determined in a certain way, reflecting the average volume of the trees, for species and geographical areas, in relation with the main factorial characteristics of the volume (diameter at breast height, height, tree form). For spruce species, first regional yield tables were calculated by Toma, the study area being Călimani (Toma, 1940). The using of mathematical models to generate yield tables was introduced in Romania in 1975 by Giurgiu and by Neamţu, and later, it was developed by Giurgiu, in 1979.

Pang defines "local" as the rule applicable to forest areas with a restricted range (Pang, 2000).

Dendrometrical elements that need to be measured to calculate the volume of standing wood are the diameter at a height of 1.3 m (diameter at breast height) and the total height of the tree. Diameter at breast height has become the standard method of expressing the standing tree trunk diameter and is the most common of dendrometrical measurements (Brack, 2009).

This paper attempts to evaluate the relation between diameter at breast height and tree hight, namely between diameter at breast height and the volume of the tree, for based on these relations, to be calculated regression coefficients for equations of height and volume.

MATERIAL AND METHOD

The study area

The study was conducted in a natural spruce stand aged 40 years, with third grade production. This stand is located in the northern Eastern Carpathians, in Vatra Dornei area, Suceava county, located at 1250 m altitude, SV exposure. The sample consists of randomly chosen felled trees, on which diameters have been measured, at the level of the ground, from meter to meter, and the total length of the tree. In the sample were measured only trees from the spruce species. 1133 diameters were measured along the tree form of 43 felled trees. The diameters were measured by the model: d_0, $d_{0,3}$, $d_{0,5}$, d_1, $d_{1,3}$, $d_{1,5}$, d_2, d_3 ... d_n. The volume of each tree was calculated using the formula of Newton.

Mathematical models

Models proposed to be studied are shown in Table 1 (equations of heights) and Table 2 (equations of volume). Thus, this paper will verify six mathematical models for calculation of heights curve, respectively 14 mathematical models to calculate the volume.

Models for height

The height of the trees is really necessary because it is used to calculate the volume. When height is not measured for each tree individually, for calculating the volume we use height equations based on the correlative link that exists between the diameter measured at different distances from the ground and the total height of the tree. Based on these relations, knowing the diameter at breast height or another diameter on the tree form, we can calculate height of the trees. Calculations have shown that each equation has a minimum diameter, after which the equation estimates negative values. Also, each equation has a maximum diameter association, so height can not be calculated for diameters larger than the maximum associate diameter or less than the minimum diameter. In Figure 1 we can see height distribution in relation to the diameters of the measured length of 1,3 m.

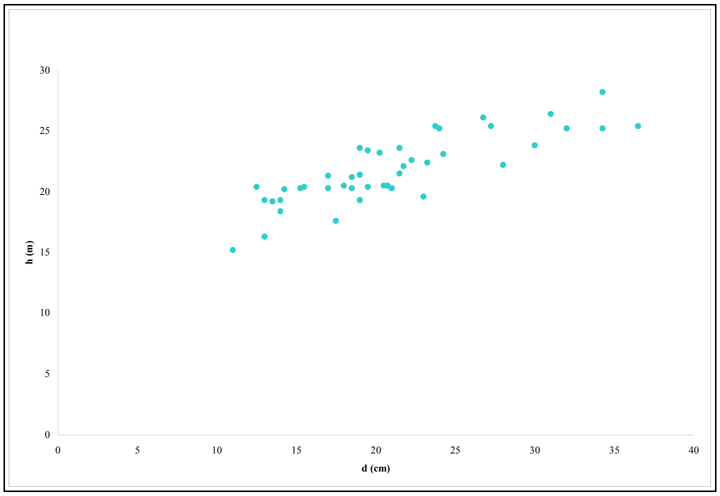

Figure 1. The relation between the diameter at breast height and the height of tree

In determining regression coefficients for equations in Table 1, statistical packages were used, made available by computer programs. The calculation was performed using linear regression through least squares method, this method being one of the most used method of calculation. For conditioned models, it was used the formula of y-intercept (regression model), where to the constant a (free term) it was assigned the value of 1.3. The values of regression coefficients, of determination coefficient, R^2 respectively of the standard error s_e are shown in Table 1.

Table 1

Mathematical models for calculating height

No.	**Equation model**	**a**	**b**	**c**	**R^2**	**s_e**
Parabola						
1	$H = a+bD+cD^2$	10,21245	72,58341	-77,273	0,713	1,5353
Conditioned parabola						
2	$H = 1,3+bD+cD^2$	151,8989	-240,058		0,993	1,7193
Hyperbola						
3	$H=a+b/D+cD$	21,17713	-0,71873	20,4507	0,713	1,5339
Conditioned hyperbola						
4	$H=1,3-bD(D+1)+cD$	240,0577	391,9566		0,993	1,7193
5	$\log H=a+b\log d+c\log^2 d$	1,528761	0,183269	-0,1343	0,706	0,0314
6	$\ln h = 1,3 + \frac{d^2}{a+bd+cd^2}$	-9,534E-05	0,004945	0,02699	0,977	0,0001

Standard error $- s_e -$ *was* calculated with the relation:

$$s_e = \sqrt{\frac{\sum (h - h_{calc})^2}{N-3}},$$

where s_e represents standard error of the measured heights from those calculated using regression, being the estimation of standard deviation of errors. Analyzing the data in Table 1 we notice that the equations 5 and 6 register the smallest deviations of the measured physical elements in the field.

Procedure for the distribution of deviations for equation 6 can be seen in Figure 2. If we refer to R^2, the coefficient of determination, it shows how much from the variation of the dependent variable is explained by the estimated equation. So, this term indicates that the estimated regression will be better as the change in estimated values will be closer to the variation of observed values.

Analyzing the R^2 is observed that the equations 2 and 4 show the greatest strength of association, which means that 99.3% of the total variation in height can be explained by the linear relation between height and diameter at breast height and only 0.7% of cases remain unexplained.

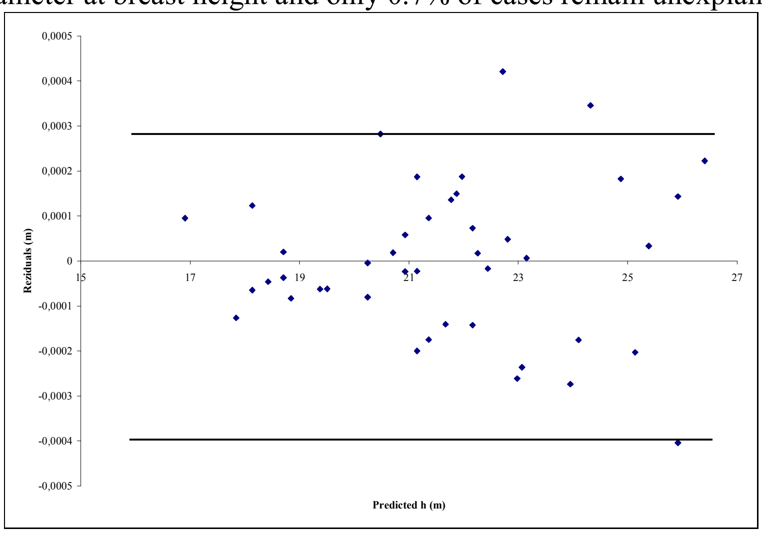

Figure 2. Rezidual plot for equation 6

As it can be seen in Figure 2, the points are concentrated in one region (residuals region with -0,0003 şi 0,0003 range values) horizontal stripe type, which does not contradict the assumptions of normality of errors. This uniform strip is characterized by a constancy of dispersion of residuals, on the entire area of the independent variable h.

Equations volume may be defined, depending on variables taken into consideration, where the general form is: V = V (h); V = V (d); V = V (dh); where V is the volume expressed in cubic meters, h is the height of the tree expressed in meters, d is the diameter at breast height, the diameter measured at 1.3 m above the ground. In Table 2 we have proposed 14 mathematical models (Giurgiu V, 1979, Avery T., 2002, Clutter J., 1983, and others).

Table 2

Mathematical models for volume calculation

No.	Equation model	a	b	c	d	e	R^2	se
Regression equations based on the diameter at breast height								
7	$V=a+bD^2$	-0,039	9,272				0,978	0,042
8	$V=a+bD+cD^2$	-0,044	0,052	9,164			0,978	0,042
9	$V=aD+bD^2$	-0,342	9,971				0,993	0,042
10	LogV=a+bLogD	1,088	2,258				0,973	0,049
Regression equations based on height								
11	$V=a+bh^2$	-0,536	0,002				0,741	0,144
12	$V=a+bh+ch^2$	1,725	-0,208	0,007			0,771	0,137
13	$V=ah+bh^2$	-0,051	0,003				0,923	0,141
14	$V=a+b\log h$	-6,627	4,605				0,781	0,140
Regression equations based on base diamter and height								
15	$V=aD^2h$	0,357					0,995	0,037
16	$V=a+bD^2h$	0,031	0,339				0,987	0,032
17	$V=D^2(a+bh)$	3,029	0,233				0,996	0,033
18	$LnV=a+bLnD+cLn(h^2/(h-1.3)$	-1,352	1,898	1,046			0,984	0,089
19	$\log v=a+b\log D+c\log h$	-0,468	1,898	0,978			0,984	0,039
20	$\log v=a+b\log D+c\log^2 D+d\log h+e\log^2 h$	-1,303	1,444	-0,337	2,044	-0,412	0,984	0,039

The regression coefficients a, b, c, d, e were calculated using the method of least squares.

Analyzing the data in Table 2 it can be observed that the equations 10 and 11 register the lowest deviations from the volumes physically measured in the field. The volume considered for reference was calculated using the formula of Newton, based on field measurements made on tree , segment measurements 1m. Regarding R^2, coefficient of determination, we observe that the equations 11 and 9 show the greatest strength of association. For equation 11, we may say that 99.6% of the total variation in volume can be explained by the regression equation based on height and diameter at breast height and only 0.4% of cases remain unexplained by this method.

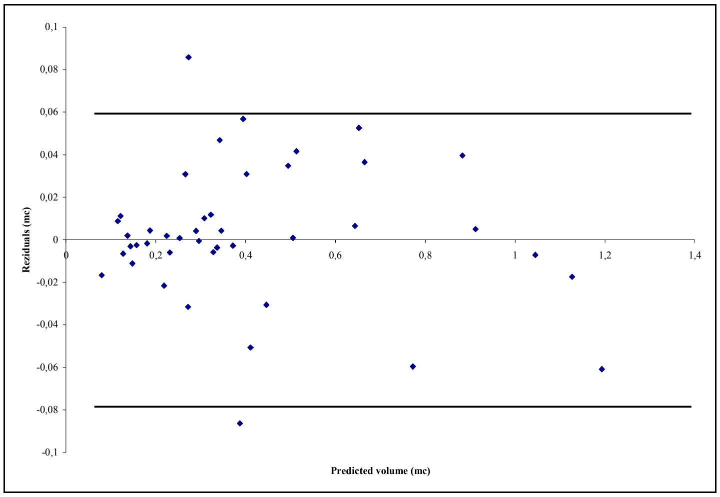

Figure 3. Rezidual plot for equation 17

Figure 3 is a graphical representation of the concentration of residuals calculated for equation 17. As it can be seen they are concentrated in a region of horizontal strip which does not contradict the assumptions of normality of errors.

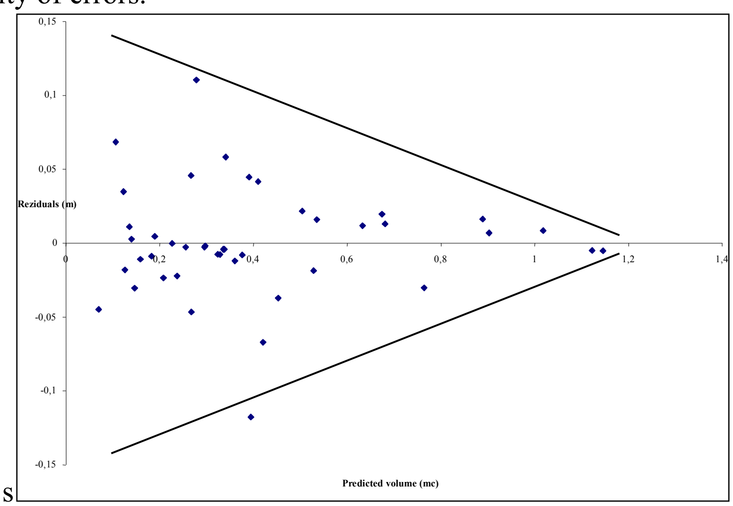

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Figure 4. Rezidual plot for equation 20

In Figure 4 was graphically represented the dispersion mode of residuals calculated for the equation 20. As it can be seen, the dispersion error is not constant, this changing according to volume values. For this mathematical model, the value of the coefficient of determination is 0,932.

Taking into account the data in the above table, we can see that the mathematical models which are based on two variables, respectively the diameter at breast height and tree height are more reliable and provide information more accurate than the mathematical models that are based on a single variable. In these models, as shown in Table 2, the mathematical models using diameter at breast height as a variable are more accurate than the mathematical models based on the variable height. The only regression equation that uses height as a variable and provided satisfactory results is the model 13 with a coefficient of determination value of 0.984 and a standard error of 0.039. On such models as shown in Table 2, using mathematical models as variable the diameter at breast height are more accurate than the mathematical models based on variable height. The only variable regression equation that uses height to provide satisfactory results is the model 13 with a lower coefficient of determination of 0.984 and a standard error of 0.039.

RESULTS AND DISSCUSIONS

As a result of this study, were chosen as models to calculate the height equation 6:

$\ln h = 1,3 + \frac{d^2}{a + bd + cd^2}$, and for volume calculation - equation 17, $V=D^2(a+bH)$.

Using regression coefficients previously calculated for both volume and height, it was calculated the height curve using equation 6 and the volume with equation 17. With these mathematical models based on measurements on land, the local regression coefficients were calculated and it has been generated a local yield table, namely Table 3.

We can see that with the increase in height there is an increase in diameter that quantifies in the accumulation of wood volume.

This work aimed to evaluate the relation between diameter at breast height and tree height, respectively diameter at breast height and the tree volume, for on the basis of these relations to be calculated the regression coefficients for equations of height and volume.

These coefficients are determined for a restricted forest area which may be used in calculating the volume through the regression equation or to calculate the local yield tables. As it can be seen in Table 2, to calculate the volume, we can use many regression equations.

The accuracy of mathematical models is determined using the statistical indicators, the correlation, the coefficient of determination respectively the standard error. Also, this study provides information on development mode of spruce, as presented, with the possibility to estimate a further development of the respective stand, as shown in Table 3.

Table 3

Local volume table for spruce

D cm	H m	V mc	D cm	H m	V mc
6	13	0,022	56	29	3,105
8	15	0,042	60	30	3,601
12	18	0,103	64	30	4,136
16	20	0,196	68	31	4,708
20	22	0,322	72	31	5,319
24	23	0,484	76	31	5,967
28	24	0,682	80	32	6,654
32	25	0,917	84	32	7,378
36	26	1,188	88	32	8,141
40	27	1,496	92	32	8,942
44	28	1,842	96	33	9,781
48	28	2,225	100	33	10,659
52	29	2,646			

CONCLUSIONS

Acknowledgments
This paper has been prepared with the financial support of the project "Quality European Doctorate - EURODOC", Contract no. POSDRU/187/1.5/S/155450, project co-financed by the European Social Fund through the Sectoral Operational Programme "Human Resources Development" 2007-2013.

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