MATHEMATICAL MODELLING FOR CHARACTERISTIC CURVES OF WIND TURBINES

Cătaş Adriana*, Dubău Călin**

*University of Oradea, Faculty of Science, Department of Mathematics and Computer Science, 1 University St., 410087 Oradea, Romania, e-mail: acatas@uoradea.ro
** University of Oradea, Faculty of Environmental Protection, 26 Gen. Magheru St., 410048 Oradea, Romania, e-mail: calin_dubau@yahoo.com

Abstract

In the present paper the authors investigated a mathematical model for a wind turbine regarding the characteristic curve, namely adimensional curves. By making use of adimensional curves can be identified operating curves for a certain turbine. Thus are established: type of turbine, type of adimensional curves and turbine size by its diameter. The mathematical model is constructed based on characteristic curve and it is proposed for output power of the wind turbine. A comparative study for different values of rapidity is also presented.

Key words: Mathematical modelling, characteristic curves, wind turbines.

INTRODUCTION

In the last years a major interest in renewable energy resources has been observed. Wind energy is a renewable energy, ecological and sustainable.

The restriction energy traditional sources (coal, oil, natural gas) and their higher prices are the classical motivation in development of renewable resources, with environmental protection by reducing greenhouse gas issue, which is now becoming as important motivation as well.

Within aero-electrical aggregates the wine turbine is the component that ensures the conversion of kinetic energy of wind into mechanical energy useable to turbine shaft, through the interaction between air current and moving blade. Wind turbine is composed mainly of a rotator fixed on a support shaft, comprising a hub and a moving blade consisting of one or more blades. Active body of aeolian turbines which made the quantity of converted energy is the blade. The achieving of aerodynamic performances, kinematics and energy curves of the aeolian turbines depend on the choice of certain geometry.

In developing of turbine blade geometry are used improved contours (airfoil) chosen and positioned so that obtained performances for certain site-specific conditions, to be optimal. The moment of interaction between pallets assembly and fluid flow comes from the lifted aerodynamic forces and resistance produced by the outline profiles. Achieving of acceptable aerodynamic efficiency requires the use of aerodynamic performance.

The performance of the wind turbine can be investigated through mathematical models and also verified by experimental measurements.

MATERIAL AND METHOD

The mathematical models designed for characteristic curves of different types and sizes of turbines represent the information used in modeling of line engine.

To determine the aerodynamic forces and aerodynamic moment it operates by formulas containing adimensional coefficients concretized in geometric and kinematic similarity relationships.

Energy performance representation that produces a wind turbine, as a whole operating area, is materialized by the characteristic curves that are operating in the optimization process. They are of two types namely: operating (exploitation) curves, respectively adimensional curves of the type of turbine (Dubau C., 2005).

There are various types of wind turbines. Between various types of wind turbines the rapid axial horizontal wind turbines are the most development ones. Many studies are also elaborated taking in consideration the turbines with vertical axes. Such a study was presented by the second author in a recent paper (Dubau C., 2009).

There was made a comparative analysis based on the results yielded by the calculations. There was compared: the vertical turbine V2500 (Gyulai F. et al., 2000 and Gyulai F., 2000 a.) and the horizontal turbine H2500 (Dubau, 2007). The adimensional curves have been constructed $C_P = f(\lambda)$, where these reference curves present the association between characteristic number λ and the maximum power value C_P (Bej A., 2003).

The exploitation curves (see Fig.1.) serve the evaluations of the annual energies, which are correlated with the areas exposed to the wind and the rotations of the turbine. We can notice from the figure that the horizontal turbine in comparison with the vertical one, accomplishes power *Parb* = 3500 at speed v =10 m/s, which is a smaller value than that of the vertical turbine, where v = 12 m/s.

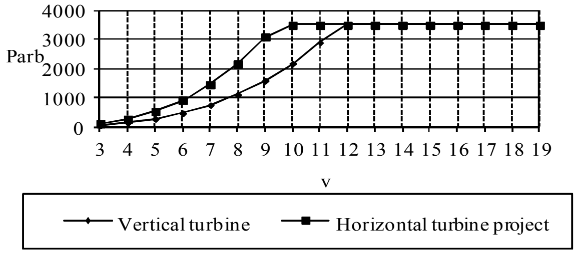

Fig.1. Operating curves

In order to characterize the functionality of various types of turbines three adimensional coefficients are used, respectively: power coefficient, moment coefficient or torque coefficients and axial force coefficient, which have the following calculation expressions (Dubau C., 2005):

$$C_P = \frac{P}{\rho \cdot \frac{v^3}{2} \cdot S}; \quad C_M = \frac{M}{\rho \cdot \frac{v^2}{2} \cdot S \cdot R}; \quad C_{F_a} = \frac{F_a}{\rho \cdot \frac{v^2}{2} \cdot S}$$

where we have used the following notations:

S – Area swept by the turbine,
R – Radius of the turbine,
P – Power turbine,
M – The moment at the engine line axis,
F_a – Axial force,
ρ – Mass density of air [kg/m^3],
v – Wind speed [m/s].

The characteristic number, namely rapidity of the turbine, is defined by the relation bellow:

$$\lambda = \frac{\pi \cdot n \cdot R}{30 \cdot v},$$

where n is the speed turbine [rpm].

In the general analysis of wind turbines of various types we have to specify several aspects regarding the rapidity and robustness of turbine.

The characteristic number associated to the turbine optimal point is denoted by λ_0 and it is called the turbine rapidity. This number characterizes the type of turbine and it represents, together with the position axis turbines, the main criteria for the characterization of the turbines. The usual field of wind turbines is $\lambda_0 = 1 \div 12$ and the turbines corresponding to the interval $\lambda_0 = 1 \div 4$ are considered “slow” in the rest of the field the turbines are considered “fast”.

RESULTS AND DISCUSSIONS

In this section we will establish the mathematical model in order to construct the adimensional curves. The proposed mathematical model is for the power output. It pursues the analytic approximation:

$$C_{PWT} = f(\lambda_0, \lambda).$$

This has a useful and easy form. Regarding the preliminaries approximations for a certain type of turbine ($\lambda_0 = ct.$) we take in considerations an individualized model which depends on maximum values of the power coefficient and its location: $(C_{PWT})_{\max}$ and λ_{0P}. It is approximated the dependence of maximum power coefficient on turbine rapidity.

$$(C_{PWT})_{\max} = f(\lambda_{0P}).$$

We will denote forward by $\lambda_0 = \lambda_{0P}$ in order to avoid the confusion with the place of maximum moment λ_{0M} which is also another characteristic of turbine type

$$(C_{PWT})_{\max} = 0,3 \cdot (\lambda_0)^{0,35} - 0,0014 \cdot (\lambda_0)^2.$$

As a preliminary approximation it is also accepted the approximation of the torque start coefficient

$$C_{M_0} = \frac{0,2}{\lambda_0^{\ 2}}.$$

The above mentioned approximations resulted by specific analysis of the information existent in specialized literature. In the future these values can be corrected and adjusted using new information which occurs from improving technology. Within this mathematical model of the turbine, the values $(C_{PWT})_{\max}$ and (C_{M_0}) appear as constants but also depending on type of turbine (λ_0).

With λ the current value of the rapidity we can write

$$C_{PWT} = C_{M_0} \cdot \lambda + a \cdot \lambda^{\alpha} - b \cdot \lambda^{\beta}.$$

The study of this curve, using experimental results from literature, implies the evaluation of dependence of the constants on type of turbine.

$$a = f(\lambda_0);$$
$$b = f(\lambda_0);$$
$$\alpha = f(\lambda_0);$$
$$\beta = f(\lambda_0)$$

These four values allow the formulation of next conditions for power output curve:

– The curve passes through the point $(\lambda_0, (C_{PWT})_{\max})$;

– The curve has an extreme (maximum) value in the above mentioned point

$$\frac{dC_{PWT}}{d\lambda} = 0, \quad \text{for} \quad \lambda = \lambda_0 .$$

The recommended value for the constants α, β are $\alpha = 2{,}0$ and $\beta = 2{,}3$.

We will use different values of the rapidity, for the constants computation, which depends on the type turbine. These values are pointed in the table below (Table 1):

Table 1

The values of constants for the power output coefficients computation

λ_0	C_{M_0}	a	b
5	0,008	0,1439	0,0777
12	0,0014	0,0269	0,0111

We plot below the power output curves of the turbines for two values of the current rapidity λ.

We can see in Fig. 2 that for $\lambda < 4$ the curve has positive values.

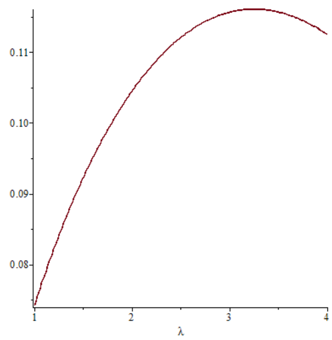

Fig. 2 Power output curve for $\lambda_0 = 5$

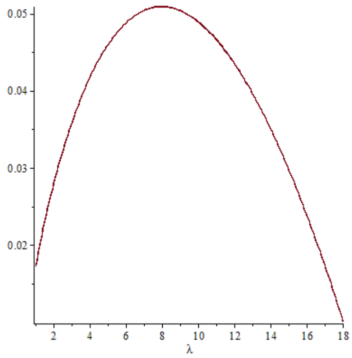

Fig. 3 Power output curve for $\lambda_0 = 12$

CONCLUSIONS

Based on plotted characteristic curves of wine turbine we can conclude that exist a major influence of type of turbine on curves shape.

Thus the above formulas for the power output curve allow a better optimization of energy performance of the wind turbines. They are also permitted a good management of the control and automation. Significant differences occur only in the small rapidity domain.

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