MODAL ANALYSIS OF RECTANGULAR WOOD PLATES WITH DIFFERENTS BOUNDARY CONDITIONS

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Abstract

In modal analysis, mechanical systems with a few inputs and hundreds of outputs have to be identified. This requires adapted frequency-domain estimators designed to handle large amount of data in a reasonable amount of time. Today, modal analysis has become a widespread means of finding the modes of vibration of a machine or structure.

Modes are inherent properties of a structure, and are determined by the material properties (mass, damping, and stiffness), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape ("modal parameters"). If either the material properties or the boundary conditions of a structure change, its modes will change.

Key words: Modal analysis, rectangular plate, boundary conditions.

INTRODUCTION

As it is known, the dynamic action request is caused by time-varying loads having the effect of moving the structure. Needless to say, the dynamic loading of a structure, balancing the instantaneous external forces, is not possible due to tensions inside the material inertia. Therefore, the unbalanced forces will drive the mass of the structure. In this case of charging structures, the determination of stress and strain, it is necessary to take into account the features of the structure stiffness and the fact that its properties depend on the inertial mass distribution in the structure. By applying dynamic external forces, the structure will begin to vibrate. Overall structures for gravity load resistance is calculated is deemed to be applied static. There are circumstances either by natural phenomena or processes in which a dynamic analysis. As a result of structural vibration, the effects of dynamic action are represented by sectional efforts, displacements, stress, and strain. Knowing the normal modes of vibration of rectangular plate, its dynamic response can be determined by applying modal analysis method.

MATERIAL AND METHOD

Modal analysis of wood plates was done at the University of Oradea in 2009.

The dynamic calculation of the plate problem made by wood considered in this paper was formulated within the assumptions of

Kirchhoff's engineering. For flat plates studied were considered as having dimensions a and b along the x axis along the y-axis. Aspect ratio of these plates are $\alpha = \frac{a}{b} = 1{,}5$. Within the modal analysis methods (Bia V., Bolotin V., Botis M.) we took into account only the symmetric modes of vibration (i, j) of the following plates: flat rectangular plate simply supported, flat rectangular plate with clamped edges and flat rectangular plate simply supported on two opposite sides and clamped on the other two. Thus, for those functions where the plate natural vibration forms to be used are:

a) doubly-clamped beam, having the shapes functions (Fetea M.)

$$G_i(x) = \left(\cosh \beta_i \frac{x}{a} - \cos \beta_i \frac{x}{a}\right) - k_i \cdot \left(\sinh \beta_i \frac{x}{a} - \sin \beta_i \frac{x}{a}\right)$$

$$G_j(y) = \left(\cosh \beta_j \frac{y}{b} - \cos \beta_j \frac{y}{b}\right) - k_j \cdot \left(\sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b}\right)$$

b) simply supported beam, having the shapes functions (Fetea M.)

$$H_i(x) = \sin \beta_i \frac{x}{a}$$

$$H_j(y) = \sin \beta_j \frac{y}{b}$$

All three types of plates were considered under the action of a uniformly distributed system of disruptive forces variable in time. The application of modal analysis was neglected damping, so the differential equation of plate movement is caused by disruptive forces (Rosca, I., Bratu, P.):

$$\nabla^4 w(x,y,t) + \frac{\mu}{D} \cdot \ddot{w}(x,y,t) = \frac{p(x,y)}{D} \cdot f(t)$$

The application of modal analysis was neglected damping, so the differential equation of plate movement is caused by disruptive forces

$$\nabla^4 w(x,y,t) + \frac{\mu}{D} \cdot \ddot{w}(x,y,t) = \frac{p(x,y)}{D} \cdot f(t)$$

Where:

The application of modal analysis was neglected damping, so the differential equation of plate movement is caused by disruptive forces:

- $w(x,y,t)$, represents the points displacements
- D, plate bending stiffness
- μ, density plate
- $\ddot{w}(x,y,t)$, second order derivative with respect to time.

The functions of vibration of plates are expressed as an infinite series:

$$W(x,y)=\sum_{i}^{\infty}\sum_{j}^{\infty}W_{ij}(x,y)=\sum_{i}^{\infty}\sum_{j}^{\infty}X_i(x)\cdot Y_j(y)$$

$$W_{ij}(x,y)=X_i(x)\cdot Y_j(y)$$

Where: $X_i(x), Y_j(y)$, are the fundamental natural vibration forms of the beams corresponding normal modes of vibration considered. As follows, the general solution is (Munteanu, Gh., Posea, N., Szilard R.):

$$w(x,y,t)=\sum_{i}^{\infty}\sum_{j}^{\infty}X_i(x)\cdot Y_j(y)\cdot\eta_{ij}(t)$$

$$\ddot{\eta}_{ij}(t)+\omega_{ij}^2\cdot\eta_{ij}(t)=\frac{L_{ij}}{m_{ij}}\cdot f(t)$$

.

Given the orthogonality property of functions the natural vibration forms of the beams, while determining the functions associated with vibrational modes *ij* is based on the equation:

For case studies performed, it was considered wood plates under the action of harmonic disturbing forces evenly distributed throughout the area having its own intensity $p(x,y)$ and frequency $\Omega=5\frac{rad}{\sec}$.

$$P=p(x,y)\cdot f(t)$$

$$f(t)=\sin\Omega t$$

The expression of dynamic multiplier function is:

$$\psi_{ij}=\frac{1}{\left|1-\left(\frac{\Omega}{\omega}\right)^2\right|}.$$

Modal and total system response can be calculated using the following relations

$$w(x,y,t)=W_{ij}(x,y)\cdot\gamma_{ij}\cdot\frac{1}{\left|1-\left(\frac{\Omega}{\omega}\right)^2\right|}$$

$$w(x,y,t)=\sum_{i}^{\infty}\sum_{j}^{\infty}W_{ij}(x,y)\cdot\gamma_{ij}\cdot\frac{1}{\left|1-\left(\frac{\Omega}{\omega}\right)^2\right|}.$$

The modal dynamic efforts of plates studied is

$$M_{ij}(x,y,t)=M_{ij}(x,y)\cdot\gamma_{ij}\cdot\psi_{ij}(t),$$

$$M_{ij}(x,y,t) = M_{ij}(x,y)\cdot\gamma_{ij}\cdot\frac{1}{\left|1-\left(\frac{\Omega}{\omega_{ij}}\right)^2\right|}.$$

Total dynamic stresses were determined by applying the principle of superposition:

$$M_{ij}(x,y,t) = \sum_{i}^{\infty}\sum_{j}^{\infty} M_{ij}(x,y)\cdot\gamma_{ij}\cdot\frac{1}{\left|1-\left(\frac{\Omega}{\omega_{ij}}\right)^2\right|}$$

RESULTS AND DISSCUSIONS

The study was carried out considering a quarter of what is on the board: $b = 3, a = 2$. For the simply supported plate the modal displacements are (Table 11,12,13,14).
By applying the principle of superposition of normal modes of vibration we considered the resulting total displacement (table 15) and the modal bending moments presented in tables 1 to 10.

$$w_{tot} = w_{11} + w_{13} + w_{31} + w_{33}$$

Table 1

M_{11}^{x}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0039	0,0068	0,0079
2	0	0,0056	0,0096	0,0111

Table 2

M_{13}^{x}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0075	0	0,0075
2	0	0,0106	0	-0,0106

Table 3

M_{31}^{x}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0159	0,0276	0,0319
2	0	-0,0225	-0,0390	-0,0450

Table 4

M_{33}^{x}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0266	0	-0,0266
2	0	-0,0376	-0,0001	0,0376

Table 5

M_{tot}^{x}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0540	0,0345	0,0056
2	0	-0,0439	-0,0294	0,0069

Table 6

M_{11}^{y}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0023	0,0040	0,0047
2	0	0,0033	0,0057	0,0066

Table 7

M_{13}^{y}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0175	0	-0,0175
2	0	0,0248	0	-0,0248

Table 8

M_{31}^{y}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0039*	*0,0068*	*0,0079*
2	*0*	*-0,0056*	*-0,0096*	*-0,0111*

Table 9

M_{33}^{y}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0157*	*0*	*-0,0157*
2	*0*	*-0,0222*	*0*	*0,0222*

Table 10

M_{tot}^{y}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0395*	*0,0109*	*-0,0207*
2	*0*	*-0,0003*	*-0,0039*	*-0,0071*

Table11

w_{11}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0.0059*	*0,0102*	*0,0117*
2	*0*	*0,0083*	*0,0144*	*0,0166*

Table 12

w_{13}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0068*	*0*	*-0,0068*
2	*0*	*0,0096*	*0*	*-0,0096*

Table 13

w_{31}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0028*	*0,0049*	*0,0057*
2	*0*	*-0,004*	*-0,007*	*-0,008*

Table 14

w_{33}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0044*	*0*	*-0,0044*
2	*0*	*-0,0062*	*0*	*0,0062*

Table 15

w_{tot}

b/a	*0*	*1*	*2*	*3*
0	*0*	*0*	*0*	*0*
1	*0*	*0,0199*	*0,0151*	*0,0062*
2	*0*	*0,0076*	*0,0074*	*0,0052*

For clamped plate the modal, total displacements and bending moments are presented in tables 16 to 30.

Table 16

w_{11}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0031	0,0083	0,0107
2	0	0,0057	0,0154	0,0197

Table 17

w_{13}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0019	0,0005	-0,0019
2	0	0,0035	0,0010	-0,0035

Table 18

w_{31}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0058	0,0155	0,0200
2	0	-0,0060	-0,0159	-0,0205

Table 19

w_{33}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0010	0,0003	-0,0010
2	0	-0,0011	-0,0003	0,0011

Table 20

w_{tot}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0118	0,0247	0,0278
2	0	0,0022	0,0001	-0,0032

Table 21

M_{11}^{x}

b/a	0	1	2	3
0	0	-0,0101	-0,0271	-0,0348
1	-0,0017	0,0006	0,0033	0,0045
2	-0,0031	0,0054	0,0176	0,0230

Table 22

M_{13}^{x}

b/a	0	1	2	3
0	0	-0,0061	-0,0017	0,0062
1	-0,0018	0,0016	0,0005	-0,0019
2	-0,0034	0,0055	0,0016	-0,0062

Table23

M_{31}^{x}

b/a	0	1	2	3
0	0	-0,0640	-0,1740	-0,220
1	-0,0106	0,0454	0,1083	0,129
2	-0,0196	-0,0351	-0,1186	-0,170

Table 24

M_{33}^{x}

b/a	0	1	2	3
0	0	-0,0113	-0,0032	0,0115
1	-0,0010	0,0075	0,0022	-0,0078
2	0,0010	-0,0086	-0,0024	0,0089

Table 25

M_{tot}^{x}

b/a	0	1	2	3
0	0	-0,0916	-0,2034	-0,2374
1	-0,0152	0,0552	0,1142	0,1240
2	-0,0250	-0,0328	-0,1018	-0,1449

Table 26

M_{11}^{y}

b/a	0	1	2	3
0	0	-0,0020	-0,0054	-0,0070
1	-0,0084	-0,0018	0,0036	0,0058
2	-0,0155	-0,0025	0,0090	0,0136

Table 27

M_{13}^{y}

b/a	0	1	2	3
0	0	-0,0012	-0,0003	0,0012
1	-0,0092	0,0050	0,0016	0,0067
2	-0,0169	0,0097	0,0031	-0,0128

Table 28

M_{31}^{y}

b/a	0	1	2	3
0	0	-0,0128	-0,0343	-0,0441
1	-0,0156	0,0042	0,0271	0,0369
2	0,0160	-0,0053	-0,0303	-0,0411

Table 29

M_{33}^{y}

b/a	0	1	2	3
0	0	-0,0023	-0,0006	0,0023
1	-0,0050	0,0040	0,0012	-0,0050
2	0,0051	-0,0043	-0,0013	0,0053

Table 30

M^{y}_{tot}

b/a	0	1	2	3
0	0	-0,0183	-0,0407	-0,0475
1	-0,0382	0,0115	0,0336	0,0310
2	-0,0112	-0,0023	-0,0195	-0,0350

Flat rectangular plate simply supported on two opposite sides and clamped on the other two have the following displacements and bending moments presented in tables 31 to 45 (Barsan G., Bors I., Fetea M.).

Table 31

w_{11}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0038	0,0103	0,0132
2	0	0,0054	0,0146	0,0187

Table 32

w_{13}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0024	0,0007	-0,0024
2	0	0,0034	0,0010	-0,0034

Table 33

w_{31}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0019	0,0052	0,0067
2	0	-0,0027	-0,0073	-0,0094

Table 34

w_{33}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,6898	0,1954	-0,7003
2	0	-0,9744	-0,2760	0,9892

Table 35

w_{tot}

b/a	0	1	2	3
0	0	0	0	0
1	0	0,0089	0,0164	0,0168
2	0	0,0051	0,0079	0,0068

Table 36

M^{x}_{11}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0021	0,0019	0,0071	0,0094
2	-0,0029	0,0027	0,0101	0,0133

Table 37

M^{x}_{13}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0023	0,0027	0,0008	-0,0032
2	-0,0033	0,0038	0,0012	-0,0045

Table 38

M^{x}_{31}

b/a	0	1	2	3
0	0	0	0	0
1	-0,001	0,0105	0,0292	0,037
2	0,0015	-0,0149	-0,0413	-0,053

Table 39

M^{x}_{33}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0007	0,0042	0,0012	-0,0044
2	0,0009	-0,0059	-0,0017	0,0062

Table 40

M^{x}_{tot}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0061	0,0193	0,0384	0,0396
2	-0,0038	-0,0143	-0,0318	-0,0383

Table 41

M^{y}_{11}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0104	-0,0020	0,0051	0,0079
2	-0,0147	-0,0028	0,0072	0,0112

Table 42

M^{y}_{13}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0116	0,0065	0,0021	-0,0086
2	-0,0165	0,0092	0,0029	-0,0121

Table 43

M_{31}^{y}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0052	0,0009	0,0077	0,0106
2	-0,0165	0,0092	0,0029	-0,0121

Table 44

M_{33}^{y}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0033	0,0025	0,0008	-0,0032
2	0,0047	-0,0036	-0,0011	0,0045

Table 45

M_{tot}^{y}

b/a	0	1	2	3
0	0	0	0	0
1	-0,0306	0,0079	0,0157	0,0068
2	-0,0190	0,0015	-0,0019	-0,0114

CONCLUSIONS

The analysis of data obtained in the paper will refer only to the total bending moments and displacements in relation to the axes x and y. As expressed in the total dynamic response of total bending moments about the axis x it follows:

- for the simply supported rectangular plate contour, the maximum bending moment about the axis x recorded in the node (1,1) is 0,054;
- for flat rectangular plate with fixed edges, the maximum bending moment about the axis x recorded in the node (3,1) mode is equal to 0.2374;
- for flat rectangular plate with fixed edges, the maximum bending moment about the axis x recorded in the node (3,1) is 0.0062. As expressed in the total dynamic response of total bending moments about the axis y it shows the following:
- in case of simply supported rectangular plate contour, the maximum bending moment in the y axis recorded in the node (1,1) is 0.0395;
- for flat rectangular plate with fixed edges, the maximum bending moment in the z axis recorded in the node (3,1) mode is equal to 0.0475;
- for flat rectangular plate with fixed edges, the maximum bending moment in the y axis is recorded in the node (0,1), being equal to -0.0306.

When expressed as total displacement dynamic response for the three plates taken into account, the maximum recorded displacement all around the board recessed the node (3,1). Maximum request is in the section including this node.

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