THE FRACTAL DIMENSION OF *QUERCUS PETRAEA* LEAVES

Ecaterina Fodor., O. Hâruța*

*University of Oradea, Faculty of Environmental Protection, 26 Gen. Magheru St., 410048 Oradea;
Romania, e-mail: ecaterina_fodor@yahoo.com

Abstract

Fractal exponent D is an appropriate shape descriptor for various outlines including the lobed leaves of Quercus petraea. The average values of D varied between 1.034 and 1.044. The frequency distribution of D corresponding to leaves collected at bottom, middle and top levels of the cronw follow approximatelly a normal distribution. Fractal exponent correlates poorly with size variables used in classic morphometry suggesting that shape and size variables segregate, shape being more conservative than size.

Key words: *Quercus petraea* leaves, fractals, size variables, correlation with fractal exponent D

INTRODUCTION

Foliar architecture is one of the main morphological features of the trees. Leaf morphology reflects the role of these organs which establish a metabolically active interface in the frame of various influences: abiotic as well as biotic, intrinsic as well as extrinsic factors. Leaves are among the most complex biological interfaces: lobed leaves of many *Quercus* species are effective convection-heat dissipaters (Vogel, 1970), the lobation enhancing the interception of direct sun light (Niklas, 1989). Deep-lobed leaves are frequent among the temperate *Quercus* species, characterized by rapid growth under seasonal temperate climate (Camarero et al., 2002). Despite the fact that oaks have rather simple shaped leaves, there is a considerable variation in size, shape and features of the blade margin (Jensen, 1990).

Shape and size are descriptive categories of variables used to characterize the morphology of an organism. A basic problem in multivariate morphometrics is that of quantifying shape differences from size differences (Rohlf & Bookstein, 1987) considering the shape to provide more reliable indications on the morphology of organisms (Somers, 1989). Intuitively, shape means geometric configuration (Marcus et al., 1996).

Shape can be defined as a connected set of points characterized by a feature space. Consequently, to represent a shape is to characterize it in terms of a set of features in such a way that it becomes possible to reconstruct the shape (da Fontoura-Costa & Cesar, 2001).

The natural geometry discovered by Benoit Mandelbrot (1977, 1982), the geometry of fractals became a general framework for the analysis of many natural phenomena, mostly for those which can be described by power laws. The term fractal comes from the latin word *fractus* meaning broken, characterizes spatial or temporal phenomena that are continuous but not differentiable: fractal structures do not have a single length scale thus displaying detail at every scale also, fractal dimension is an non-integer.

Biological systems and processes are typically characterized by many levels of substructure, with same general pattern being repeated in an ever-decreasing cascade (Kenkel and Walker, 1996)

Most complex biological interfaces including leaves are described by fractal exponents, this geometry enhancing the exchange or flux of matter and energy at these contact zones (Frontier, 1987). Fractal geometry offers simple shape descriptors for leaves (Campbell, 1996) and are appropriate descriptors for repetitive structures, lobation of leaves being an exemple of such because of their similarity in structure and growth rates (McLellan and Dengler, 1995).

The fractal dimension is related to shape index (ratio of area and perimeter of a closed contour) and is a reliable measure of the leaf outline complexity (Camarero et al., 2002). The distinctive feature of the fractal approach is the emphasis on self-similarity (Prusinkiewicz and Lindenmayer, 1990). An object is self similar if it can be decomposed into smaller copies of itself (Hastings and Sugihara, 1994).

The aim of the present study was to asses shape complexity of the leaf outline using the information contained in the fractal exponent for intact leaves and partially consumed by foliivorous insects, the estimation of the frequency distribution of fractal exponents D, to test the possible correlation between fractal dimension and size variables of the leaves and to compare the fractal exponents of leaves at different levels in the crown with the aid of One Way ANOVA..

MATERIALS AND METHODS

The study was located in a broadleaved recreational forest, in a hilly area, in the proximity of city of Cluj, central Romania, in stands dominated by stone oaks (*Quercus petraea*), Forestry district Cluj Napoca, Production Unit IV. Site type corresponds to hilly stone oak forests with grasses and *Luzula* sp. The tree stratum contains also: *Carpinus betulus, Acer campestre, Tilia corda.* The understorey is established by *Evonymus europea, Rosa canina* and seedlings. During the observations period recursive attacks of *Melolontha melolontha* (Coleoptera:Scarabeidae) occurred.

The leaves were collected in upper, middle and bottom crown level of a tree vegetating in the interior forest. Groups of one hundred intact leaves were sampled from the indicated crown levels and separately, those partially consumed (a total amount of six hundred leaves) during the month of August, years 2005 and 2006.

Shape complexity and the variation of shape depending on the position at different crown levels were assessed using fractal dimension D. Box-plot method was employed to calculate the fractal exponents with HARFA package. The collected leaves were scanned after herborisation using Mustek 1200CP scanner, at 600dpi. The outline of every leaf was obtained digitally for fractal analysis. Area, perimeter, the length of the long axis, the length of the mediolateral axis were estimated on the digital image (1 pixels=0.4233 cm). Important morphometric ratios were calculated: the ratio between area and perimeter and the ratio between long and mediolateral axes.

Box counting method is currently used to measure the fractal dimension of a curve by superimposing a regular grid of pixels of length δ on the object to be studied and by counting the number of occupied pixels (c). The method is based on the box-counting dimension, a classical approach to fractal analysis in image processing (Fontoura-Costa & Cesar, 2001). It consists in covering a shape as the leaf outline with boxes and finding how the number of boxes changes with the size of the boxes.

This procedure is repeated using different values of δ. The power-law relationship in this case is (Kenkel & Walker, 1996):

$$C=K\,\delta^{-D}$$

where: D- fractal exponent of dimension, C – number of occupied pixels, δ - Length of the unit box, k – constant

Because there are frequently re-orientation of the grid that produce different values of c, grid placement replications are needed in order to obtain a distribution of D values. In the present study the employed δ were of 2; 4 and 6 pixels.

Calculated fractal exponents were employed for the construction of frequency distribution and this was tested for normality using kurtosis and skewness criteria. The comparison of estimated D exponents quantifying the shape of leaves at different levels in the crown as well as between the group of intact leaves and the group of incomplete leaves was achieved using one way ANOVA and Tukey's multiple comparison tests. Also, a multiple comparison among all groups of leaves, intact and consumed was performed.

Simple regression was employed to asses the possible correlation of D with most important size variables of the leaves: area, perimeter, the length of the long axis, the length of the mediolateral axis and the ration between area/perimeter and long axis/mediolateral axis All statistical tests were performed using the KyPlot package version 2.0 beta.

RESULTS

The relatively low average D values for different levels in the crown and in intact and consumed leaves are summarized in table1.

Table 1

Average fractal dimension, average perimeter, average area and the coefficient of variation of intact and consumed leaves, at three crown levels collected from a tree (*Quercus petraea*) vegetating in the forest interior of a mixed stand (30-40 yr.).

Leaves/crown level	**Mean fractal exponent ±SD**	**R2**	**area±SD cm2**	**CV**	**perimeter±SD cm**	**CV**
bottom crown, intact leaves	1.0355±0.0104	0.99	27.55±17.49	0.63	27.96±10.01	0.35
middle level, intact leaves	1.0367±0.0113	0.99	27.05±15.96	0.58	27.82±10.19	0.36
top level intact leaves	1.0371±0.0104	0.99	27.77±13.90	0.50	28.15±8.45	0.30
bottom level, consumed leaves	1.044±0.0167	0.99	20.78±15.59	0.75	27.86±12.00	0.43
middle level, consumed leaves	1.0419±0.0209	0.99	16.14±13.16	0.81	23.49±11.93	0.50
top level, consumed leaves	1.034±0.0182	0.99	13.62±0.50	0.77	21.63±9.88	0.45

Table 2

Skewness and kurtosis of the estimated fractal exponent D of *Quercus petraea* leaves (each sample consisting of 100 leaves)

Location in the tree crown/leaves' integrity	**skewness**	**Kurtosis**	**Comments**
bottom level, intact leaves	-0.1593	-0.12599 platykurtic	Sspread of data toward higher values and high variability
top level, intact leaves	0.2577	0.6225 leptokurtic	Spread of data toward smaller D values and low variability
middle level, intact leaves	-0.2223	0.8872 leptokurtic	Spread of data toward smaller D values
bottom level, consumed leaves	0.0597	1.5431 leptokurtic	Spread of data toward higher D values
Middle level, consumed leaves	0.2072	-0.3157 platykurtic	Spread of data toward smaller D values and low variability
top level, consumed leaves	-0.0030	-0.2470 platykurtic	Spread of data toward higher D values and low variability

One way ANOVA and Tukey's multiple comparisons test failed to find significant differences among fractal exponents D calculated at three levels in the crown in the case of intact leaves. The leaves which were partially consumed by insects, however presented very significant differences among groups (F_{calc}.=7.1735, $F_{0.05}$=3.0361)

The result was confirmed by Tukey's test in the following order: between bottom and middle group of leaves the differences were not significant at a $P \leq 0.05$, but there were extremely significant differences between the bottom and the top groups of leaves ($P \leq 0.001$) and significant differences between the middle and top group of leaves ($P \leq 0.05$). An overall comparison was performed using Tukey's test for all groups of leaves, at all tree levels, both intact and consumed (table 4).

The correlation was tested between D and other morphometric descriptors of the leaves, applied only to intact leaves as being typical at the species level (table 3).

Table 3

Size variables of *Quercus petraea* leaves at different levels in the crown and their simple regressions on fractal dimension D

Position in the crown	**Variables (average±SD)**						
Bottom intact leaves	1.Area (cm^2)	2.Perimeter (cm)	3.Long axis (cm)	4.MD axis(cm)	5.A/P	6.L/l	7.Fractal D
	27.55±17.49	27.96±10.01	8.17±2.46	4.74±1.65	0.89±0.26	1.76±0.17	1.03±0.01
	Regression statistics						
	R^2	R	R^2_{adj}	a	b	T test for a	T test for b
	(1&7) 0.0908	0.3014	0.0815	1.0305	0.0017	***	***
	(2&7) 0.1057	0.3251	0.0966	1.0260	0.0003	***	***
	(3.&7) 0.1282	0.3580	0.1193	1.0230	0.0015	***	***
	(4&7) 0.0913	0.3022	0.0820	1.0265	0.0018	***	**
	(5&7) 0.0857	0.2927	0.0763	1.0250	0.0111	***	**
	(6&7) 0.0022	-0.0477	-0.0110	1.040	-0.0028	***	NS
	Variables (average±SD)						
Middle intact leaves	Area (cm^2)	Perimeter (cm)	Long axis (cm)	MD axis(cm)	A/P	L/l	Fractal D
	27.05±15.96	22.82±10.19	7.94±2.39	4.77±1.57	0.89±0.24	1.69±0.24	1.03±0.01
	Regression statistics						
	R^2	R	R^2_{adj}	a	b	T test for a	T test for b
	(1&7) 0.0527	0.2297	0.0431	1.0323	0.0001	***	*
	(2&7) 0.0724	0.2692	0.0630	1.0284	0.0003	***	**
	(3&7) 0.0456	0.2137	0.0359	1.0287	0.0010	***	*
	(4&7) 0.0748	0.2736	0.0654	1.0270	0.0019	***	**
	(5&7) 0.0478	0.2187	0.0301	1.0277	0.0100	***	*
	(6&7) 0.0241	-0.1555	0.0142	1.0419	-0.0071	***	NS
	Variables (average±SD)						
Top intact leaves	Area (cm^2)	Perimeter (cm)	Long axis (cm)	MD axis(cm)	A/P	L/l	Fractal D
	20.77±13.90	28.58±8.45	7..96±2.0	5.01±1.54	0.92±0.23	1.62±0.18	1.03±0.01
	Regression statistics						
	R^2	R	R^2_{adj}	a	b	T test for a	T test for b
	(1 &7) 0.0253	0.1592	0.0154	1.0338	0.0001	***	NS
	(2&7) 0.0289	0.1702	0.019	1.0312	0.0002	***	NS
	(3&7) 0.0305	0.1749	0.0207	1.0299	0.0009	***	NS
	(4&7)0.0182	0.1351	0.0082	1.0325	0.0009	***	NS
	(5&7) 0.1231	0.0151	0.0051	1.0321	0.0054	***	NS
	(6&7) 0.0004	0.0219	-0.0097	1.035	-0.0012	***	NS

Significances tested over all possible combinations of intact and consumed leaves at different levels in the crown show a great variability of the significance level.

Table 4

Significance levels for multiple comparisons among groups of *Q. petraea* leaves at three crown levels, intact and consumed.

Comparisons	**Significance**
1+2;1+3;1+6;2+3;2+5;2+6;3+5;+3+6;4+5;	Not significant, NS
1+4;4+6	***, extremely significant P≤0.0001
2+4;3+4	**, very significant P≤0.05
1+5;5+6	*, significant P≤0.05

1= bottom complete leaves; 2=middle complete leaves; 3= top complete leaves; 4=bottom consumed leaves; 5= middle consumed leaves; 6= top consumed leaves

DISCUSSIONS AND CONCLUSIONS

Leaf shapes are related to factors such as leaf support, supply and shading of other leaves. The shading depends on growth habit and internode length. All these factors together with genetic and environmental contribute to the leaf shape being included in a phenotypic space. Fitness of a particular leaf shape must be considered as a consequence, in a higher dimensional space that incorporate additional features to size and shape (Langlade et al., 2005). Fractal dimension of a leaf estimated with box-counting method is a good descriptor of the outline rugosity including lobation. The reported fractal dimension of *Q. petraea* leaves using the same box-counting method, collected from seedlings (Camarero et al., 2002) is 1.12 to 1.13, higher values as compared to mature trees leaves in our study. The maximum D in *Quercus* group was reported by same authors for *Q. coccifera* (1.15) and the smallest for *Q. velutina* (1.06). For *Q. rubra* is given an average value of 1.08 for D calculated using the compass method (Borkowski, 1999). However, D is not a good descriptor when discriminating between spiny and deeply lobed leaves in the same genus, namely *Quercus* (*Q. coccifera* versus *Q. pyrenaica*) according to Camarero et al., (2003). An interesting fact is observed analyzing average values of D in intact and consumed leaves (table 1), values which vary in a small interval: the consumption does not induce great modification in leaf overall geometry because consumption does not create more rugosity of the perimeter.

The frequency distribution of the calculated D approaches normal distribution. Skewness and kurtosis of the data (table 2) display small values and the tendency toward higher D values and higher variability of the data are observed at intact leaves of the bottom and consumed leaves of bottom and top levels. Both statistics indicate that more sampling is needed to confirm normal distribution. Both intact and consumed leaves show the same tendency toward normal distribution (of the same fractal exponent) meaning that consumption modifies at random the shape of leaves. We presume according to the normal distribution of D that there is an average shape of leaves of the same species corresponding to tree age, genetic traits, position within the stand and site conditions (intrinsic and extrinsic factors) and D is a primary descriptor of this shape. Further approaches may offer more information on the average shape and a promising method appears to be Elliptic Fourier Transform (Yoshioka et al., 2004).

As it is expected, the differences among D calculated for intact and consumed leaves are significant due to random variability induced by consumption which modifies drastically the shape of the leaf. However, no significant differences were found among groups of leaves at different crown levels with regard to D. Overall multiple comparisons (Tukey test) revealed several combinations, mostly involving consumed leaves which are significantly different (table 4) suggesting that consumption determines different patterns of the leaf outline at different crown levels. Size variables do not vary significantly among crown levels (unpublished data) but vary within groups of leaves (within group variability greater than between group variability).

The fractal exponents correlate poorly with other morphometric descriptors such as leaf area, leaf perimeter, long axis, mediolateral axis, area/perimeter and long axis/mediolateral axis ratios (table 6) although these dimensions correlate highly among them. This fact emphasizes the significance of the fractal exponent which is related to the shape complexity rather than to the leaf metric dimensions. Other workers demonstrated that another shape descriptor, the ratio between leaf area and perimeter correlates better with fractal exponent (Camarero et al., 2003) although our results do not confirm this statement.

REFERENCES

1. Borkowsi, W., 1999, Fractal dimension based features are useful descriptors of leaf complexity and shape. Can J. For. Res. **29**: 1301-1310.
2. Camarero, J.J., S. Sisó, E. Gil-Pelegrin, 2002, Fractal dimension does not adequately describe the complexity of leaf margin in seedlings of Quercus species. Anales Jard. Bot. Madrid **60**(1): 63-71.
3. Campbell, R.D., 1996, Fractal geometry offers simple descriptors. Acta biotheoretica **44**(2): 19-142.
4. Da Fontura-Costa, L., M.R. jr Cesar, 2001, Shape Analysis and Classification. Theory and Practice. CRC Press, Boca Raton, London, New York, Washington D.C.
5. Frontier, S. 1987, Application of fractal theory to ecology. In Legendre, P., Legendre, L. (eds.). Developments in numerical ecology, Berlin: 335-378.
6. Hastings, H.M., G. Sugihara, 1994, Fractals. A user's guide for the natural sciences. Oxford Science Publications. Oxford University Press.
7. Jensen, R.H., 1990, Detecting shape variation in oak leaf morphology: a comparison of Rotational Fit Methods. American Journal of Botany **77**(10): 1279-1293.
8. Kenkel, N.C., D.J. Walker, 1996, Fractals in biological sciences. Coenoses **11**: 77-100.
9. Kenkel, N.C., D.J. Walker, 1996, Fractals in the biological sciences. Quantitative Plant Ecology Laboratory. Coenoses **11**: 77-100.
10. Langlade, N.B., F. Xianzhong, T. Dransfield, L. Copsey, A. I. Hanna, C. Thébaud, A. Bangham, A. Hudson, E. Coen, 2005, Evolution through genetically controlled allometry space. PNAS **102**(29): 10221-10226.
11. Mandelbrot, B., 1977, *Fractals: Form, Chance and Dimension*. Flammarion, Paris
12. Mandelbrot, B., 1982, *The fractal geometry of nature*. Freeman, San Francisco.
13. Marcus, L.F., M. Corti, A. Loy, G.P.J. Naylor, D.E. Slice, 1996, *Advances in morphometrics. NATO Series*. Advanced Science Institute Series. Plenum Press. New York, London.
14. McLellan, T., N.G. Dengler, 1995, Pattern and form in repeated elements in the development of simple leaves of *Begonia dregei*. *Int. J. Plant Sci.* **156**(5): 581-589.

15. Niklas, K.J., 1989, The effect of lobbing on the interception of direct solar radiation. *Oecologia* **80**:59-64.
16. Prusinkiewicz, P., A. Lindenmayer, 1990, *The algorithmic beauty of plants*. The virtual laboratory. Springer Verlag, New York.
17. Rohlf, F.J., F.L. Bookstein, 1987, A comment on shearing as a method for size correction. *Systematic Zoology* **36**(4): 356-367.
18. Somers, K.M., 1989, Allometry, Isometry and shape in Principal Component Analysis. *Systematic Zoology* **38**(2): 189-173.
19. Vogel, S., 1970, Connective cooling at lobe airspeeds and the shapes of broad leaves. *J. Exp. Bot.* **21**: 81-100.
20. Yoshioka, Y., H. Iwata, R. Ohsawa, S. Ninomyia, 2004, Analysis of petal shape variation of *Primula siebholdii* by Elliptic Fourier Descriptors and Principal Component Analysis. *Annals of Botany* **94**(5): 657-664.